CHAPTER 3

EXPONENTS AND RADICALS

INTRODUCTION

Do you remember exponents of rational numbers and their laws? Let us recall.

- Zero exponent → [For any non-zero rational number a, we define $a^{o} = 1$]

- Negative integral exponent → [Let a be any non-zero rational number and n be a positive integer, then we define $a^{-n} = \frac{1}{a^{n}}$]

- Laws of exponents → [Let a and b be any two rational numbers and let m and n be integers, then we have

  (i) $a^{m} \times a^{n} = a^{m+n}$ (ii) $\frac{a^{m}}{a^{n}} = a^{m-n}, a \neq 0$

  (iii) $(a^{m})^{n} = a^{mn} = (a^{n})^{m}$ (iv) $(ab)^{n} = a^{n}b^{n}$

  (v) $\left(\frac{a}{b}\right)^{n} = \left(\frac{a^{n}}{b^{n}}\right), b \neq 0$]

In this Chapter, we shall study more about rational exponents and their laws.

RATIONAL NUMBERS AS EXPONENTS

We have already read about squares, square roots, cubes and cube roots in detail in the previous chapters. Now, we know that

$4^{2} = 16$ and inversely $\sqrt{16} = 4$,

i.e. 4 raised to the power 2 is 16 and square root of 16 is 4.

or we can write $\sqrt{16} = 4$ as $16^{\frac{1}{2}} = 4$ (sixteen raised to the power half is four)

Here, 16 is the base and $\frac{1}{2}$ is the exponent.

Similarly, $4^3 = 64$ and inversely $\sqrt[3]{64} = 4$

or $(64)^{\frac{1}{3}} = 4$

When a quantity has its exponent in the form of a fraction (rational number), its numerator denotes the exponent (power) of the quantity while its denominator denotes the radical (root) of the quantity, i.e.

(i) $(225)^{\frac{3}{2}}$ means square root of $(225)^3$.

(ii) $(216)^{\frac{2}{3}}$ means cube root of $(216)^2$.

or In general, $a^{\frac{m}{n}}$ means the n^{th} root of a^m.

If a be any positive rational number and m be an integer while n be a natural number, then $a^{\frac{m}{n}} = \sqrt[n]{a^m}$.

Here, a^m is called the **radicand**, $\sqrt[n]{a^m}$ is called a **radical** and n is called the **index** of the radical. The sign $\sqrt{}$ is called the **radical sign**.

Note: Index of a radical is always a natural number.

Example 1: Express the following in exponential form.

(i) $\sqrt{7}$ (ii) $\sqrt[3]{21}$ (iii) $\sqrt[5]{265}$

(iv) $\sqrt[3]{3^2}$ (v) $\sqrt[8]{\left(\frac{5}{6}\right)}$

Solution: (i) $\sqrt{7} = 7^{\frac{1}{2}}$ (ii) $\sqrt[3]{21} = (21)^{\frac{1}{3}}$ (iii) $\sqrt[5]{265} = (265)^{\frac{1}{5}}$

(iv) $\sqrt[3]{3^2} = (3^2)^{\frac{1}{3}} = 3^{\frac{2}{3}}$ (v) $\sqrt[8]{\left(\frac{5}{6}\right)} = \left(\frac{5}{6}\right)^{\frac{1}{8}}$

Example 2: Express the following as radicals.

(i) $(5)^{\frac{1}{3}}$ (ii) $(17)^{\frac{2}{5}}$ (iii) $\left(\frac{12}{19}\right)^{\frac{3}{5}}$

Solution: (i) $(5)^{\frac{1}{3}} = \sqrt[3]{5}$ (ii) $(17)^{\frac{2}{5}} = \sqrt[5]{(17)^2}$ (iii) $\left(\frac{12}{19}\right)^{\frac{3}{5}} = \sqrt[5]{\left(\frac{12}{19}\right)^3}$

Positive Rational Numbers as Exponents

Let a be any positive rational number and $x\left(\text{i.e. } \frac{m}{n}\right)$ be a positive rational exponent.

Then, we define $a^x\left(\text{i.e. } a^{\frac{m}{n}}\right)$ as the n^{th} root of a^m.

Similarly,

$$4^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8$$

$$9^{\frac{3}{2}} = (9^3)^{\frac{1}{2}} = (729)^{\frac{1}{2}} = 27$$

i.e.

In general, $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$

Example 3: Find the value of the following:

(i) $(125)^{\frac{2}{3}}$ (ii) $8^{\frac{4}{3}}$

Solution:

(i) $(125)^{\frac{2}{3}} = [(125)^2]^{\frac{1}{3}}$ $\qquad \because a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}}$

$= \sqrt[3]{125^2} = \sqrt[3]{125 \times 125}$

$= \sqrt[3]{\underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}} = 5 \times 5 = 25$

Thus, the value of $(125)^{\frac{2}{3}} = 25$.

(ii) $8^{\frac{4}{3}} = (8^4)^{\frac{1}{3}} = \sqrt[3]{8^4}$

$= \sqrt[3]{8 \times 8 \times 8 \times 8} = 8\sqrt[3]{8} = 8\sqrt[3]{\underline{2 \times 2 \times 2}}$

$= 8 \times 2 = 16$

Thus, the value of $8^{\frac{4}{3}} = 16$.

Negative Rational Numbers as Exponents

Look carefully at the following:

$$4^{\frac{-3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{(4^3)^{\frac{1}{2}}} = \frac{1}{(64)^{\frac{1}{2}}} = \frac{1}{8}$$

$$9^{\frac{-3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{(729)^{\frac{1}{2}}} = \frac{1}{27}$$

In general, let a be a non-zero positive rational number and $\frac{m}{n}$ be a positive rational number in the lowest form, then we define

$$a^{\frac{-m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{(a^m)^{\frac{1}{n}}}$$

Example 4: Find the values of each of the following:

(i) $(27)^{\frac{-2}{3}}$ (ii) $(343)^{\frac{-2}{3}}$ (iii) $(512)^{\frac{-2}{9}}$

Solution:

(i) $$(27)^{\frac{-2}{3}} = \frac{1}{(27)^{\frac{2}{3}}} = \frac{1}{(27^2)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27^2}} = \frac{1}{\sqrt[3]{27\times 27}}$$

$$= \frac{1}{\sqrt[3]{\underline{3\times 3\times 3}\times \underline{3\times 3\times 3}}} = \frac{1}{3\times 3} = \frac{1}{9}$$

Thus, the value of $(27)^{\frac{-2}{3}} = \frac{1}{9}$.

(ii) $$(343)^{\frac{-2}{3}} = \frac{1}{(343)^{\frac{2}{3}}} = \frac{1}{(343^2)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{343\times 343}}$$

$$= \frac{1}{\sqrt[3]{\underline{7\times 7\times 7}\times \underline{7\times 7\times 7}}} = \frac{1}{7\times 7} = \frac{1}{49}$$

Thus, the value of $(343)^{\frac{-2}{3}} = \frac{1}{49}$.

(iii) $$(512)^{\frac{-2}{9}} = \frac{1}{(512)^{\frac{2}{9}}} = \frac{1}{(512^2)^{\frac{1}{9}}} = \frac{1}{\sqrt[9]{512^2}} = \frac{1}{\sqrt[9]{512\times 512}}$$

$$= \frac{1}{\sqrt[9]{\underline{4\times 4\times 4\times 4\times 4\times 4\times 4\times 4\times 4}}} = \frac{1}{4}$$

Thus, the value of $(512)^{\frac{-2}{9}} = \frac{1}{4}$.

Worksheet 1

1. Express each of the following in exponential form.

(i) $\sqrt[5]{35}$ (ii) $\sqrt[11]{(27)^2}$ (iii) $\sqrt[7]{\dfrac{11}{3}}$ (iv) $\sqrt[3]{\left(\dfrac{2}{5}\right)^{-3}}$

(v) $\sqrt[13]{(111)^3}$ (vi) $\sqrt[7]{(29)^2}$ (vii) $\sqrt[3]{(2)^{-6}}$ (viii) $\sqrt[7]{\left(\dfrac{15}{341}\right)^{-3}}$

2. Express each of the following as radicals.

(i) $(21)^{\frac{1}{8}}$ (ii) $(25)^{\frac{3}{4}}$ (iii) $\left(\dfrac{2}{9}\right)^{\frac{1}{9}}$ (iv) $(100)^{\frac{-5}{6}}$

(v) $\left(\dfrac{8}{9}\right)^{\frac{3}{4}}$ (vi) $\left(\dfrac{17}{231}\right)^{\frac{-5}{6}}$ (vii) $\left(\dfrac{15}{21}\right)^{\frac{2}{5}}$

3. Express each of the following with positive exponent.

(i) $x^{\frac{-1}{2}}$ (ii) $x^{\frac{-2}{5}}$ (iii) $\dfrac{7}{x^{\frac{-5}{6}}}$ (iv) $(x^{-3})^4$

Laws of Rational Exponents

To understand laws of exponents for rational exponents, let us do some examples.

Example 5: Find the value of $\left(\dfrac{4}{9}\right)^{\frac{3}{2}} \times \left(\dfrac{4}{9}\right)^{\frac{1}{2}}$ and $\left(\dfrac{4}{9}\right)^{\frac{4}{2}}$. Are they equal?

Solution: First, consider $\left(\dfrac{4}{9}\right)^{\frac{3}{2}} \times \left(\dfrac{4}{9}\right)^{\frac{1}{2}}$

$$= \left[\left(\frac{2}{3}\right)^2\right]^{\frac{3}{2}} \times \left[\left(\frac{2}{3}\right)^2\right]^{\frac{1}{2}} = \left(\frac{2}{3}\right)^{2\times\frac{3}{2}} \times \left(\frac{2}{3}\right)^{2\times\frac{1}{2}}$$

$$= \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^1 = \left(\frac{2}{3}\right)^{3+1} = \left(\frac{2}{3}\right)^4$$

$$= \frac{(2)^4}{(3)^4} = \frac{16}{81} \qquad \ldots\text{(i)}$$

Now, consider $\left(\dfrac{4}{9}\right)^{\frac{4}{2}} = \left[\left(\dfrac{2}{3}\right)^2\right]^{\frac{4}{2}}$

$$= \left(\frac{2}{3}\right)^{2\times\frac{4}{2}} = \left(\frac{2}{3}\right)^{4} = \frac{(2)^4}{(3)^4} = \frac{16}{81} \qquad \ldots \text{(ii)}$$

From (i) and (ii),

$$\left(\frac{4}{9}\right)^{\frac{3}{2}} \times \left(\frac{4}{9}\right)^{\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{4}{2}}$$

Law I: If x is any rational number and a, b are rational exponents, then $x^a \times x^b = x^{a+b}$.

Example 6: Are the values of $\left(\frac{64}{125}\right)^{\frac{2}{3}} \div \left(\frac{64}{125}\right)^{\frac{5}{3}}$ and $\left(\frac{64}{125}\right)^{\frac{2}{3}-\frac{5}{3}}$ equal?

Solution: First, consider $\left(\frac{64}{125}\right)^{\frac{2}{3}} \div \left(\frac{64}{125}\right)^{\frac{5}{3}}$

$$= \left(\frac{4^3}{5^3}\right)^{\frac{2}{3}} \div \left(\frac{4^3}{5^3}\right)^{\frac{5}{3}}$$

$$= \left[\left(\frac{4}{5}\right)^{3}\right]^{\frac{2}{3}} \div \left[\left(\frac{4}{5}\right)^{3}\right]^{\frac{5}{3}} = \left(\frac{4}{5}\right)^{3\times\frac{2}{3}} \div \left(\frac{4}{5}\right)^{3\times\frac{5}{3}}$$

$$= \left(\frac{4}{5}\right)^{2} \div \left(\frac{4}{5}\right)^{5} = \left(\frac{4}{5}\right)^{2-5} = \left(\frac{4}{5}\right)^{-3}$$

$$= \left(\frac{5}{4}\right)^{3} = \frac{125}{64} \qquad \ldots \text{(i)}$$

Now, consider $\left(\frac{64}{125}\right)^{\frac{2}{3}-\frac{5}{3}} = \left(\frac{64}{125}\right)^{\frac{-3}{3}}$

$$= \left(\frac{64}{125}\right)^{-1} = \frac{125}{64} \qquad \ldots \text{(ii)}$$

From (i) and (ii),

$$\left(\frac{64}{125}\right)^{\frac{2}{3}} \div \left(\frac{64}{125}\right)^{\frac{5}{3}} = \frac{125}{64} = \left(\frac{64}{125}\right)^{\frac{2}{3}-\frac{5}{3}}$$

Law II: If x be any rational number ($x > 0$), a and b are rational exponents then $x^a \div x^b = x^{a-b}$.

Example 7: Evaluate $\left[\left(\frac{36}{25}\right)^{\frac{3}{2}}\right]^{\frac{5}{3}}$ and $\left(\frac{36}{25}\right)^{\frac{3}{2}\times\frac{5}{3}}$

Solution : First, consider $\left[\left(\frac{36}{25}\right)^{\frac{3}{2}}\right]^{\frac{5}{3}}$

$$= \left[\left(\frac{6^2}{5^2}\right)^{\frac{3}{2}}\right]^{\frac{5}{3}} = \left(\frac{6^2}{5^2}\right)^{\frac{3}{2}\times\frac{5}{3}}$$

$$= \left(\frac{6}{5}\right)^{2\times\frac{3}{2}\times\frac{5}{3}}$$

$$= \left(\frac{6}{5}\right)^{5} = \frac{(6)^5}{(5)^5} = \frac{7776}{3125} \quad \text{...(i)}$$

Now, consider $\left(\frac{36}{25}\right)^{\frac{3}{2}\times\frac{5}{3}}$

$$= \left(\frac{36}{25}\right)^{\frac{5}{2}}$$

$$= \left(\frac{6^2}{5^2}\right)^{\frac{5}{2}} = \left[\left\{\left(\frac{6}{5}\right)^2\right\}^{\frac{1}{2}}\right]^5 = \left(\frac{6}{5}\right)^5$$

$$= \frac{(6)^5}{(5)^5} = \frac{7776}{3125} \quad \text{... (ii)}$$

From (i) and (ii),

$$\left[\left(\frac{36}{25}\right)^{\frac{3}{2}}\right]^{\frac{5}{3}} = \frac{7776}{3125} = \left(\frac{36}{25}\right)^{\frac{3}{2}\times\frac{5}{3}}$$

Law III: If x ($x > 0$) is a rational number and a, b are rational exponents, then $(x^a)^b = x^{ab} = (x^b)^a$.

Example 8: Verify whether $(64)^{\frac{2}{3}} \times (27)^{\frac{2}{3}}$ and $(64 \times 27)^{\frac{2}{3}}$ are equal?

Solution: Consider $(64)^{\frac{2}{3}} \times (27)^{\frac{2}{3}}$

$$= \left(4^3\right)^{\frac{2}{3}} \times \left(3^3\right)^{\frac{2}{3}} = 4^2 \times 3^2$$

$$= 16 \times 9 = 144 \qquad \text{... (i)}$$

Now, consider $(64 \times 27)^{\frac{2}{3}}$

$$= (1728)^{\frac{2}{3}}$$

$$= (12^3)^{\frac{2}{3}} = \left(12^3\right)^{\frac{2}{3}} = 12^2 = 144 \qquad \text{... (ii)}$$

From (i) and (ii),

$$(64)^{\frac{2}{3}} \times (27)^{\frac{2}{3}} = (64 \times 27)^{\frac{2}{3}}$$

Law IV: If $x, y > 0$ are rational numbers and a is rational exponent, then $x^a \times y^a = (xy)^a$.

Example 9: Verify whether $\left(\frac{16}{81}\right)^{\frac{1}{2}}$ and $\frac{16^{\frac{1}{2}}}{81^{\frac{1}{2}}}$ are equal?

Solution: $$\left(\frac{16}{81}\right)^{\frac{1}{2}} = \left[\left(\frac{4}{9}\right)^2\right]^{\frac{1}{2}} = \frac{4}{9}$$

and $$\frac{16^{\frac{1}{2}}}{81^{\frac{1}{2}}} = \frac{4^{2\left(\frac{1}{2}\right)}}{9^{2\left(\frac{1}{2}\right)}} = \frac{4}{9}$$

Thus, $$\left(\frac{16}{81}\right)^{\frac{1}{2}} = \frac{16^{\frac{1}{2}}}{81^{\frac{1}{2}}}$$

Law V: If $x, y > 0$ are rational numbers, where y ≠ 0 and a is a rational exponent, then

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}.$$

Let us now solve some examples on laws of exponents.

Example 10: Find the value of each of the following:

(i) $6^{\frac{1}{2}} \times 6^{\frac{3}{2}}$ (ii) $3 \times 16^{\frac{3}{4}}$ (iii) $2 \times 27^{-\frac{2}{3}}$

Solution:

(i) $6^{\frac{1}{2}} \times 6^{\frac{3}{2}} = 6^{\frac{1}{2}+\frac{3}{2}}$ $\quad \because x^a \times x^b = x^{a+b}$

$$= 6^{\frac{1+3}{2}} = 6^{\frac{4}{2}} = 6^2 = 36$$

(ii) $3 \times 16^{\frac{3}{4}} = 3 \times (2^4)^{\frac{3}{4}}$

$$= 3 \times 2^{4 \times \frac{3}{4}} = 3 \times 2^3 \qquad \because (x^a)^b = x^{ab}$$

$$= 3 \times 8 = 24$$

(iii) $2 \times 27^{-\frac{2}{3}} = 2 \times (3^3)^{\frac{-2}{3}}$

$$= 2 \times 3^{3 \times \frac{-2}{3}} = 2 \times 3^{-2} \qquad \text{[using } (x^a)^b = x^{ab}\text{]}$$

$$= \frac{2}{3^2} = \frac{2}{9} \qquad \left(x^{-m} = \frac{1}{x^m}\right)$$

Example 11: Simplify $(27)^{\frac{6}{5}} \div (27)^{\frac{1}{5}}$

Solution: Using $x^a \div x^b = x^{a-b}$, we have,

$$(27)^{\frac{6}{5}} \div (27)^{\frac{1}{5}} = (27)^{\frac{6}{5}-\frac{1}{5}}$$

$$= (27)^{\frac{5}{5}} = 27$$

Example 12: Evaluate:

(i) $(0.000064)^{\frac{5}{6}}$ (ii) $\left\{\left[(625)^{\frac{-1}{2}}\right]^{\frac{-1}{4}}\right\}^2$

Solution:

(i) $(0.000064)^{\frac{5}{6}}$

$$= \left(\frac{64}{1000000}\right)^{\frac{5}{6}} = \frac{(64)^{\frac{5}{6}}}{(1000000)^{\frac{5}{6}}}$$ $\left[\text{using } \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}\right]$

$$= \frac{(2^6)^{\frac{5}{6}}}{(10^6)^{\frac{5}{6}}} = \frac{2^{6\times\frac{5}{6}}}{10^{6\times\frac{5}{6}}}$$ $\because (x^a)^b = x^{ab}$

$$= \frac{2^5}{10^5} = \frac{32}{100000} = 0.00032$$

(ii) $$\left\{\left[(625)^{\frac{-1}{2}}\right]^{\frac{-1}{4}}\right\}^2$$

$$= \left\{\left[(25^2)^{\frac{-1}{2}}\right]^{\frac{-1}{4}}\right\}^2 = \left\{\left[25^{2\times\frac{-1}{2}}\right]^{\frac{-1}{4}}\right\}^2$$ [using $(x^a)^b = x^{ab}$]

$$= \left[(25^{-1})^{\frac{-1}{4}}\right]^2 = \left(25^{-1\times\frac{-1}{4}}\right)^2 = \left(25^{\frac{1}{4}}\right)^2$$

$$= 25^{\frac{1}{4}\times 2} = 25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5$$

Example 13: Solve each of the following exponential equations.

(i) $7^x = 343$ (ii) $2^{x-3} = 1$

Solution:

(i) $7^x = 343$

$\Rightarrow$ $7^x = 7^3$

When bases are the same, we equate powers.

Therefore, $x = 3$

(ii) $2^{x-3} = 1$

$\Rightarrow$ $2^{x-3} = 2^o$

When bases are the same, we equate powers.

$\Rightarrow$ $x - 3 = 0$

Therefore, $x = 3$

Worksheet 2

1. Simplify:

(i) $x^{\frac{1}{2}} \times x^{\frac{5}{2}}$ (ii) $\dfrac{x^{\frac{6}{5}}}{x^{\frac{1}{5}}}$ (iii) $(x^7)^0$

(iv) $5x^{\frac{5}{6}} \times 6x^{\frac{1}{6}}$ (v) $x^{-\frac{7}{2}} \times 2x^{-\frac{1}{2}}$

2. Find the value of:

(i) $(512)^{\frac{-2}{9}}$ (ii) $[(216)^{\frac{2}{3}}]^{\frac{1}{2}}$ (iii) $10 \div 8^{\frac{-1}{3}}$

(iv) $(16)^{\frac{3}{4}}$ (v) $27^{\frac{1}{3}} \times 16^{\frac{-1}{4}}$ (vi) $\dfrac{1}{[(3^4)^{\frac{1}{2}}]^{-2}}$

(vii) $\dfrac{27^{\frac{-2}{3}} \times 81^{\frac{5}{4}}}{\left(\frac{1}{3}\right)^{-3}}$ (viii) $64^{\frac{1}{2}}(64^{\frac{1}{2}} + 1)$ (ix) $\dfrac{36^{\frac{7}{2}} - 36^{\frac{9}{2}}}{36^{\frac{5}{2}}}$

(x) $4 \times 81^{\frac{-1}{2}}(81^{\frac{1}{2}} + 81^{\frac{3}{2}})$

3. Evaluate:

(i) $(0.04)^{\frac{3}{2}}$ (ii) $(6.25)^{\frac{3}{2}}$ (iii) $(0.03125)^{\frac{-2}{5}}$ (iv) $(0.008)^{\frac{2}{3}}$

4. Evaluate:

(i) $(6^2 + 8^2)^{\frac{1}{2}}$ (ii) $[5(8^{\frac{1}{3}} + 27^{\frac{1}{3}})^3]^{\frac{1}{4}}$

(iii) $(17^2 - 8^2)^{\frac{1}{2}}$ (iv) $(1^3 + 2^3 + 3^3)^{\frac{-5}{2}}$

5. Simplify and express the answers with positive indices.

(i) $2x^{\frac{1}{6}} \times 2x^{\frac{-7}{6}}$ (ii) $\left[\sqrt[4]{\left(\frac{1}{x}\right)^{-12}}\right]^{\frac{-2}{3}}$ (iii) $a^{\frac{4}{7}} \div a^{\frac{10}{7}}$

6. **Verify that—**

$$\left[(729)^{\frac{-5}{3}}\right]^{\frac{-1}{2}} = (729)^{\frac{-5}{3}\times\left(-\frac{1}{2}\right)}$$

7. **Solve the given exponential equations.**

(i) $\left(\sqrt{6}\right)^{x-2} = 1$ (ii) $3^{4x} = \frac{1}{81}$ (iii) $(\sqrt{2})^{x} = 2^{8}$ (iv) $2^{2x+1} = 4^{2x-1}$

Value Based Question

As part of 'Earth Day' celebrations an interclass poster making competition was arranged between nine sections of Class–VIII. Each section was provided a sheet measuring 75 cm × 40 cm for making the poster.

(a) Find the total area of sheet provided to the students. Express the area in exponential form.

(b) Write any two ways by which you can save earth.

Brain Teasers

1.A. Tick (✓) the correct option.

(a) Value of $\left\{(625)^{\frac{-1}{2}}\right\}^{2}$ is—

(i) $\frac{1}{625}$ (ii) – 25 (iii) – 625 (iv) 25

(b) If $5^x = 1$, value of x is—

(i) 5 (ii) $\frac{1}{25}$ (iii) 0 (iv) $\frac{1}{5}$

(c) Value of $\left(27^{\frac{1}{3}} + 64^{\frac{1}{3}}\right)^{2}$ is—

(i) $7^{\frac{1}{3}}$ (ii) 49 (iii) $7^{\frac{1}{2}}$ (iv) $\frac{1}{7}$

(d) For any two non-zero rational numbers a and b, $a^4 \div b^4$ is equal to—

(i) $(a \div b)^1$ (ii) $(a \div b)^0$ (iii) $(a \div b)^4$ (iv) $(a \div b)^8$

(e) 4.25×10^{-7} is equal to—

(i) 0.425000000 (ii) 4250000 (iii) 0.000000425 (iv) 0.00000425

B. Answer the following questions.

(a) Express $\left(\frac{13}{21}\right)^{\frac{2}{5}}$ as a radical.

(b) Find the value of $(81)^{\frac{-3}{4}}$

(c) Simplify $5 \times 16^{\frac{3}{4}}$

(d) Evaluate $(0.000064)^{\frac{5}{6}}$

(e) Solve for x if $3^{x-1} = \frac{1}{27}$

2. Simplify : $\dfrac{(64)^{\frac{-1}{6}} \times (216)^{\frac{-1}{3}} \times (81)^{\frac{1}{4}}}{(512)^{\frac{-1}{3}} \times (16)^{\frac{1}{4}} \times (9)^{\frac{-1}{2}}}$

3. Simplify and express the answer with positive exponent: $\left[\sqrt[3]{x^4 y} \times \dfrac{1}{\sqrt[3]{xy^7}}\right]^{-4}$

4. Evaluate:

(i) $3 \times (16)^{\frac{3}{4}}$ (ii) $2 \times (27)^{\frac{-2}{3}}$ (iii) $2 \times 9^{\frac{3}{2}} \times 9^{\frac{-1}{2}}$

5. Find the value of $[(5)^2 + (12)^2]^{\frac{1}{2}}$.

6. Find the value of x, if:

(i) $2^x + 2^x + 2^x = 192$ (ii) $8^{255} = (32)^x$ (iii) $2^{2x+2} = 4^{2x-1}$

7. If $4^x - 4^{x-1} = 24$, then find the value of x.

HOTS

1. **Evaluate : $(6^{-1} - 8^{-1})^{-1} + (2^{-1} - 3^{-1})^{-1}$**

2. **If $3^{x-1} = 9$ and $4^{y+2} = 64$, find the value of $\frac{y}{x} - \frac{x}{y}$.**

Enrichment Questions

1. **If $9^x \times 3^2 \times \left(3^{\frac{-x}{2}}\right)^{-2} = \frac{1}{27}$, find x.**

2. **By what number should $\left(\frac{-3}{2}\right)^{-3}$ be divided so that the quotient may be $\left(\frac{-8}{27}\right)^{-2}$.**

You Must Know

1. If m is a positive integer greater than 1, and a, b are rational numbers such that $b^m = a$, then we write $a^{\frac{1}{m}} = b$.

2. $a^{\frac{1}{m}}$ is called the m^{th} root of a and may be written as $\sqrt[m]{a}$.

3. For any positive integer m and n, and any non-zero rational number a, we define

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ and } a^{-\frac{m}{n}} = \sqrt[n]{a^{-m}} = \frac{1}{\sqrt[n]{a^m}}$$

4. If a is any rational number different from zero and x, y are any rational numbers, then

   (i) $a^x \times a^y = a^{x+y}$ (ii) $a^x \div a^y = a^{x-y}$ (iii) $(a^x)^y = a^{xy}$

5. If $a = \sqrt[n]{b} = b^{\frac{1}{n}}$, then $b^{\frac{1}{n}}$ is the exponential form and $\sqrt[n]{b}$ is the radical form of a, n is the index of the radical and b is the radicand.

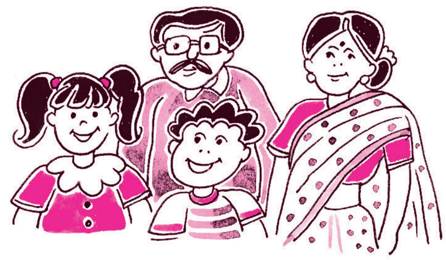
CHAPTER 4 DIRECT AND INVERSE VARIATION

INTRODUCTION

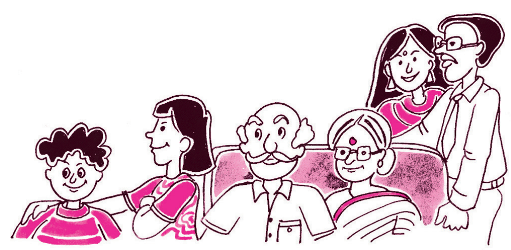
Let us consider some situations:

Situation I:

2 litres of milk is consumed by 4 members of a family.

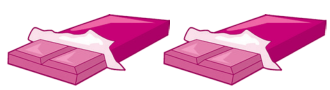


Here, 3 litres of milk is consumed by 6 members of a family.

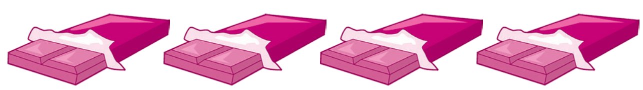


This shows that the increase in the number of members, increases the consumption of milk.

Situation II:



2 chocolates cost ₹ 10.



Therefore, 4 chocolates will cost ₹ 20.

Here, an increase in the number of chocolates results in an increase in the money spent on it.

Situation III:

With a speed of 40 km/hr, a distance of 1200 km is covered in 30 hrs.

When the speed is increased to 60 km/hr, a distance of 1200 km is covered only in 20 hrs.

This shows that increase in the speed causes a decrease in the time (distance remaining the same).

Situation IV:

For doing a particular work, 16 workers require 30 days but 20 workers require 24 days only.

Thus, an increase in the number of workers decreases the number of days (work remaining the same).

From the above examples, it is clear that:

If the values of two quantities are related in such a way that a change in one results in a corresponding change in the other, the two quantities are said to be in variation.

TYPES OF VARIATION

Variation is of two types:

1. Direct Variation
2. Inverse Variation

Direct Variation

Remember

If two quantities are related in such a way that an **increase** in one quantity results in a **corresponding increase** in the other and vice-versa, then such a variation is called **direct variation**.

Let us consider the following table showing the number of stamps (each of same value) denoted by the letter x and the corresponding costs denoted by the letter y.

x (Number of stamps)	5	10	15	20	25	30
y (Cost in rupees)	10	20	30	40	50	60

If you examine the table, you will find that there is an increase in the cost corresponding to the increase in number of stamps. Hence, it is a case of **direct variation**.

Let us now find the ratios of different number of stamps to the corresponding cost.

$$\frac{5}{10}, \frac{10}{20}, \frac{15}{30}, \frac{20}{40}, \frac{25}{50}, \frac{30}{60}$$

After simplifying each of them, each ratio reduces to $\frac{1}{2}$.

In other words, $\frac{x}{y}$ does not change; it remains constant.

Let $\quad \dfrac{x}{y} = k$ (constant)

$\Rightarrow \quad \boxed{x = ky}$ ... (i)

As k does not change,

From (i), x and y increase and decrease together.

Two quantities x and y are said to be in direct variation if x and y increase or decrease together in such a way that $\dfrac{x}{y}$ remains constant (positive).

Example 1: Find p and q in the following table, if x and y vary directly.

x	5	p	10
y	8	32	q

Solution: It is given that x and y vary directly.

Therefore, $\dfrac{x}{y} = k$ (constant)

$$\frac{5}{8} = \frac{p}{32} = \frac{10}{q}$$

Taking $\dfrac{5}{8} = \dfrac{p}{32}$

$$p = \frac{5 \times 32}{8}$$

$$\boxed{p = 20}$$

Taking $\dfrac{5}{8} = \dfrac{10}{q}$

$$q = \frac{8 \times 10}{5}$$

$$\boxed{q = 16}$$

Example 2: Verify whether x and y vary directly with each other. If so, find 'k'.

x	2	3	5	6
y	6	9	17	20

Solution: From the given table, x and y increase or decrease together but ratio $\dfrac{x}{y}$ for corresponding values of x and y is not constant.

As $\quad \dfrac{2}{6} = \dfrac{1}{3}$ and $\dfrac{3}{9} = \dfrac{1}{3}$

but $\frac{5}{17} \neq \frac{1}{3}$ and also $\frac{6}{20} \neq \frac{1}{3}$

$\therefore$ x and y do not vary directly.

Note: If two quantites x and y increase (or decrease) together, it does not ensure that it is a case of direct variation.

If they increase (or decrease) together, then we should try, if possible, to obtain the relation $x = ky$ (i.e. $\frac{x}{y} = k$). If we can find k, then x and y are in direct variation.

Example 3: Cost of six mineral water bottles is ₹ 90. How many mineral water bottles can be bought for ₹ 225?

Solution: Let the number of bottles for ₹ 225 = x. Then, we have,

Number of mineral water bottles	6	x
Cost (in rupees)	90	225

This is a case of **direct variation**. As the money spent on mineral water bottles increases, the number of bottles will definitely increase in the same ratio.

$$\therefore \quad \frac{6}{90} = \frac{x}{225}$$

$$x = \frac{6 \times 225}{90}$$

$$x = 15$$

$\therefore$ 15 mineral water bottles can be bought for ₹ 225.

Example 4: A man deposited a sum of ₹ 5,000 in a bank and earned an interest of ₹ 600 in two years. How much interest would a deposit of ₹ 8,000 earn in the same period? (Rate of interest being the same.)

Solution: Let interest earned on ₹ 8,000 = ₹ x.

Thus, we have,

Money deposited (in ₹)	5,000	8,000
Interest earned (in ₹)	600	x

This is a case of **direct variation**.

The money deposited and interest earned for the same period will increase in the same ratio.

$$\therefore \quad \frac{5,000}{600} = \frac{8,000}{x}$$

$$\therefore \quad x = \frac{8,000 \times 600}{5,000}$$

$$\therefore \quad x = 960$$

∴ Interest on ₹ 8,000 would be ₹ 960.

Worksheet 1

1. Fill in the missing terms in the following tables, if *x* and *y* vary directly.

(i)

x	6	8	12	—
y	18	—	—	63

(ii)

x	10	—	30	46
y	5	10	—	—

(iii)

x	6	8	10	—
y	15	20	—	40

2. Five bags of rice weigh 150 kg. How many such bags of rice will weigh 900 kg?

3. The cost of 3 kg of sugar is ₹ 105. What will be the cost of 15 kg of sugar?

4. A motor boat covers 20 km in 4 hrs. What distance will it cover in 7 hrs (speed remaining the same)?

5. A motor bike travels 210 km on 30 litres of petrol. How far would it travel on 7 litres of petrol?

6. If 12 women can weave 15 metres of cloth in a day, how many metres of cloth can be woven by 20 women in a day?

7. If the weight of five sheets of paper is 20 g, how many sheets of the same paper would weigh 2.5 kg?

8. Reena types 600 words in four minutes. How much time will she take to type 3,150 words?

9. Rohan takes 14 steps in covering a distance of 2.8 m. What distance would he cover in 150 steps?

10. **A dealer finds that 48 refined oil cans (of 5 litres each) can be packed in eight cartons of the same size. How many such cartons will he require to pack 216 cans?**

11. **The total cost of 15 newspapers is ₹ 37.50. Find the cost of 25 newspapers.**

12. **A labourer gets ₹ 675 for nine days work. How many days should he work to get ₹ 900?**

Inverse Variation

Remember

If two quantities are related in such a way that an **increase** in one causes **corresponding decrease** in the other and vice-versa, then such a variation is called an **inverse variation**.

Let us consider the following table showing the speed of the car and time taken to cover the same distance.

x (Speed in km/hr)	30	40	60	90
y (Time in hrs)	12	09	06	04

From the table given here, it is clear that as the speed of the car increases, time taken to cover the same distance decreases or vice-versa.

Did you notice?

$$30 \times 12 = 360$$
$$40 \times 9 = 360$$
$$60 \times 6 = 360$$
$$90 \times 4 = 360$$

i.e. the product of different speeds and corresponding time always comes to be the same.

In other words, xy does not change; it remains constant, i.e. $xy = k$ (constant).

Two quantities x and y are said to be in inverse variation, if an increase in x causes corresponding decrease in y (and vice-versa) in such a manner that xy remains constant. We can find a constant k such that $xy = k$.

Example 5: In the following table, x and y vary inversely. Find the value of p and q.

x	5	10	q
y	8	p	2

Solution: Since x and y vary inversely,

$\therefore$ Product xy = constant and this is equal to $5 \times 8 = 40$

$$10 \times p = 40 \quad \text{and} \quad q \times 2 = 40$$

$$\Rightarrow \quad p = 4 \qquad\qquad q = 20$$

Example 6: If 52 men can do a piece of work in 35 days. In how many days will 28 men complete the same work?

Solution: Let 28 men do the same work in x days.

$\therefore$ We have,

Number of men	52	28
Days	35	x

Clearly, as the number of men decreases, the number of days will increase.

So, it is a case of **inverse variation**.

Hence, $52 \times 35 = 28 \times x$

$$\Rightarrow \quad x = \frac{52 \times 35}{28}$$

$$\Rightarrow \quad x = 65$$

$\therefore$ 28 men will do the work in 65 days.

Example 7: In a camp, there is enough provision for 500 students for 30 days. If 100 more students join the camp, for how many days will the provision last now?

Solution: Let the number of days = x.

Now, number of students in the camp = 500 + 100 = 600

Thus, we have,

Number of students	500	600
Number of days	30	x

We note that more is the number of students, the less will be the number of days the provision will last for.

So, it is a case of **inverse variation**.

$$\therefore \quad 500 \times 30 = 600 \times x$$

$$\Rightarrow \quad x = \frac{500 \times 30}{600}$$

$$\Rightarrow \quad x = 25$$

Hence, the provision will last for 25 days.

Example 8: Rachit starts his journey to a certain place by car at 9 a.m. and reaches the place at 1 p.m., if he drives the car at a speed of 30 km/hr. By how much should he increase the speed so that he can reach the place by 12 noon?

Solution: Time taken at a speed of 30 km/hr = 9 a.m. to 1 p.m. = 4 hrs.

Desired time taken to reach the place = 9 a.m. to 12 noon = 3 hrs.

Let the desired speed of the car = x km/hr

$\therefore$ We have,

Time (in hrs)	4	3
Speed (in km/hr)	30	x

We note that more the speed, less will be the time taken to cover the given distance.

$\therefore$ It is a case of **inverse variation**.

$$4 \times 30 = 3 \times x$$

$$\Rightarrow \quad x = \frac{4 \times 30}{3}$$

$$\Rightarrow \quad x = 40$$

Hence, Rachit should drive at a speed of 40 km/hr to reach the place at 12 noon.

$\therefore$ He should increase the speed by (40 – 30) = 10 km/hr

Worksheet 2

1. In the following tables, *a* and *b* vary inversely. Fill in the missing values.

(i)

a	7	—	28
b	8	4	—

(ii)

a	2.5	4	0.5
b	8	—	—

(iii)

a	10	—	12
b	6	15	—

2. The science teacher asked the students of Class-VIII to make a project report on pollution. When 10 students work on it, the work gets finished in three days. How many students are required so that work finishes in two days?

3. Running at an average speed of 40 km/hr, a bus completes a journey in $4\frac{1}{2}$ hours. How much time will the return journey take if the speed is increased to 45 km/hr?

4. **Disha cycles to her school at an average speed of 12 km/hr. It takes her 20 minutes to reach the school. If she wants to reach her school in 15 minutes, what should be her average speed?**
5. **If 15 men can repair a road in 24 days, then how long will it take nine men to repair the same road?**
6. **If 30 goats can graze a field in 15 days, then how many goats will graze the same field in 10 days?**
7. **A contractor with a work force of 420 men can complete a work of construction of a building in nine months. Due to request by the owners he was asked to complete the job in seven months. How many extra men he must employ to complete the job?**
8. **Uday can finish a book in 25 days if he reads 18 pages every day. How many days will he take to finish it, if he reads 15 pages every day?**
9. **A shopkeeper has enough money to buy 40 books, each costing ₹ 125. How many books he can buy if he gets a discount of ₹ 25 on each book?**
10. **Six pumps working together empty a tank in 28 minutes. How long will it take to empty the tank if four such pumps are working together?**
11. **A train moving at a speed of 75 km/hr covers a certain distance in 4.8 hours. What should be the speed of the train to cover the same distance in 3 hours?**
12. **A garrison of 120 men has provision for 30 days. At the end of five days, five more men joined them. How many days can they sustain on the remaining provision?**

TIME AND WORK, TIME AND DISTANCE

As you know that the—

- amount of **work done** by a person **varies directly** with the **time taken** by him/her to complete it.
- number of persons performing a particular work **varies inversely** with the **time taken** by them.
- **speed** of a moving body and **distance** travelled by it in a particular time **vary directly** with each other.
- **speed** of a moving body and **time taken** to cover a particular distance **vary inversely** with each other.
- **distance travelled** and **time taken** by a moving body (speed remaining constant) **vary directly** with each other.

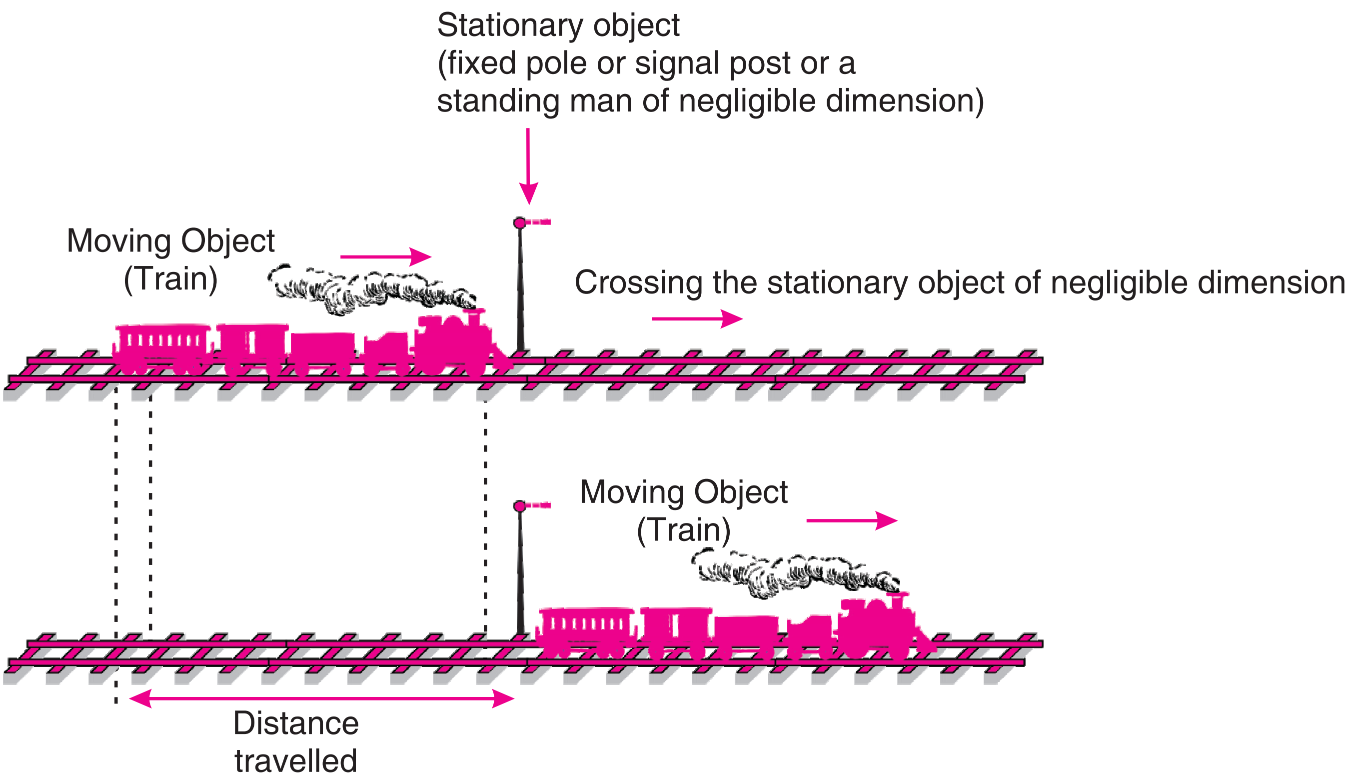
Note: The problems on time and work as well as time and distance can be solved using the concepts of direct and inverse variations.

In solving the problems of time and distance, the relation between speed, distance and time is of great importance.

Remember

Speed $= \dfrac{\text{Distance}}{\text{Time}}$	Time $= \dfrac{\text{Distance}}{\text{Speed}}$	Distance = Speed $\times$ Time

Now look at the following figure:



From the above, it is clear that:

Distance travelled by a moving object to pass a stationary object of negligible dimension	=	Length of the moving object

Example 9: A train 400 m long is running at a speed of 72 km/hr. How much time does it take to cross a telegraph post?

Solution: The length of train is given in metres. Hence, we shall convert its speed in m/sec.

$$\text{Speed} = 72 \text{ km/hr} = \frac{72 \times 1000}{3600} \qquad \left[\because \begin{array}{l} 1 \text{ km} = 1000 \text{ m} \\ 1 \text{ hr} = 3600 \text{ sec.} \end{array}\right]$$

$$= 20 \text{ m/sec.}$$

To cross a telegraph post, the train will cover a distance equal to its length, i.e. 400 m. Let the required time = x sec.

Thus, we have,

Distance (in m)	20	400
Time (in sec.)	1	x

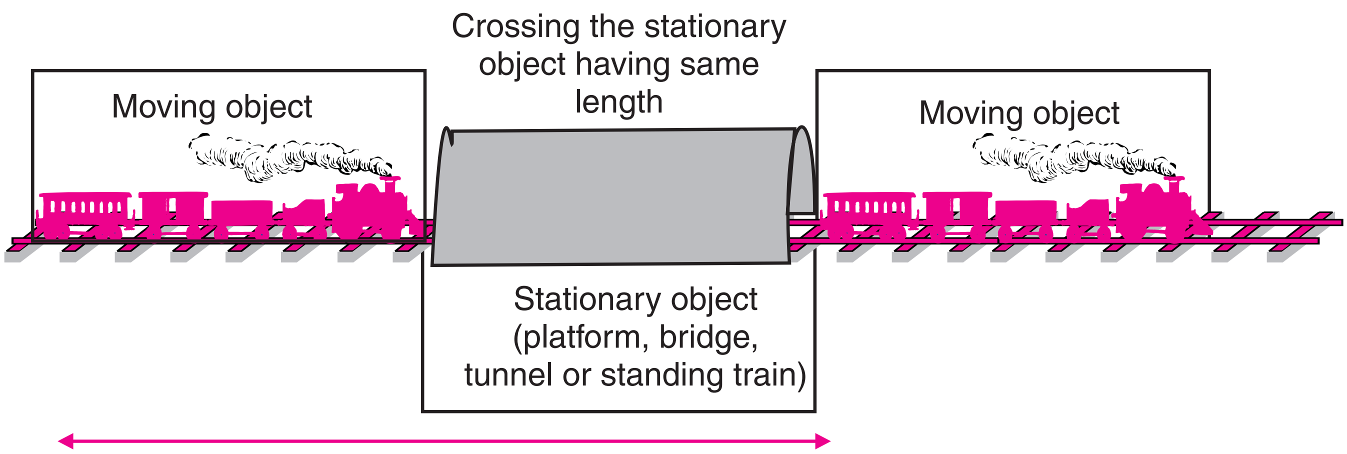
Clearly, it is a case of **direct variation**.

$$\therefore \qquad \frac{20}{1} = \frac{400}{x}$$

$$\Rightarrow \qquad x = \frac{400}{20} = 20$$

Hence, time taken by the train to cross the telegraph post is 20 seconds.

Consider the following figure now:



From the diagram, it is clear that:

Distance travelled by moving object to pass a stationary object of some length	=	Length of the moving object + Length of the stationary object

Example 10: A train 360 m long is running at a speed of 45 km/hr. What time will it take to cross a 140 m long bridge?

Solution : The length of the train is in metres.

$$\therefore \quad \text{Speed} = 45 \text{ km/hr} = \frac{45 \times 1000}{3600} \text{ m/s}$$

$$= \frac{25}{2} \text{ m/s} = 12.5 \text{ m/s}$$

To cross the bridge, the train will have to cover

$$360 + 140 = 500 \text{ m.}$$

Let required time to cross a bridge be x sec.

Thus, we have,

Distance (in m)	12.5	500
Time (in sec.)	1	x

Clearly, it is a case of **direct variation**.

$$\therefore \quad \frac{12.5}{1} = \frac{500}{x}$$

$$\Rightarrow \quad x = \frac{500}{12.5}$$

$$\Rightarrow \quad x = 40$$

$\therefore$ Train will take 40 seconds to cross a 140 m long bridge.

Worksheet 3

1. **Ramit can finish his work in 25 days, working eight hours a day. If he wants to finish the same work in 20 days, how many hours should he work in a day?**

2. **Udit can complete his work in 10 days. What amount of work will be completed in eight days?**

3. **20 men can build a wall in nine days, then how long would it take 12 men to build the same wall?**

4. **Geetika weaves 20 baskets in 30 days. In how many days she will weave 120 baskets?**

5. **A train 280 metres long is running at a speed of 42 km/hr. How much time will it take to pass a man standing on a platform?**

6. **A train 350 metres long crosses an electric pole in 28 seconds. Find the speed of the train in km/hr.**

7. **A train 150 m long is running at 72 km/hr. It crosses a bridge in 13 seconds. Find the length of the bridge.**

8. **How long will a train, 120 m long, take to clear a platform, 130 m long, if its speed is 50 km/hr?**

9. **A train 210 m long took 12 seconds to pass a 90 m long tunnel. Find the speed of the train.**

10. **A train 270 m long is running at 80 km/hr. How much time will it take to cross a platform 130 m long?**

Value Based Questions

1. **Two children were eating pizza at "Master Bakers" having three slices of pizza each. All of a sudden one of their friends came and they shared pizza slices equally among three.**

   (a) How many slices of pizza would each get?

   (b) Is it good to eat junk food like pizza? Give reason.

2. **By walking for 30 minutes in the morning, Namita covers two kilometres.**

   (a) How much distance will she cover in 20 minutes by walking at the same pace?

   (b) What is the importance of morning walk?

Brain Teasers

1.A. Tick (✓) the correct option.

(a) Both x and y are in direct proportion, then $\frac{1}{x}$ and $\frac{1}{y}$ are—

(i) in direct proportion

(ii) in inverse proportion

(iii) neither in direct nor in inverse proportion

(iv) sometimes in direct and sometimes in inverse proportion

(b) If two quantities x and y vary inversely with each other, then—

(i) $\frac{x}{y}$ remains constant (ii) $(x - y)$ remains constant

(iii) $(x + y)$ remains constant (iv) (xy) remains constant

(c) Both p and q vary directly with each other. When p is 10 and q is 15, which of the following is not a possible pair of corresponding values of p and q?

(i) 15 and 20 (ii) 20 and 30 (iii) 2 and 3 (iv) 5 and 7.5

(d) If x and y vary directly with each other and $x = 24$, what is y when constant of variation is 3?

(i) 21 (ii) $\frac{1}{8}$ (iii) 8 (iv) 27

(e) Which of the quantities vary inversely with each other?

(i) Distance travelled and cab fare.

(ii) Area of a land and its cost.

(iii) Number of workers and amount of work done.

(iv) Number of workers and time taken to finish a job.

B. Answer the following questions.

(a) l varies directly as m and l is equal to 5, when $m = \frac{2}{3}$. Find l when $m = \frac{16}{3}$.

(b) A bowler throws a cricket ball at a speed of 36km/hr. How long does this ball take to travel a distance of 20 metres to reach the batsman?

(c) Sweets from a packet were distributed among 50 children and each of them received four sweets. If it is distributed among 40 children, how many sweets will each child get?

(d) If y is directly proportional to $\frac{1}{x}$ and $x = 2$ when $y = 20$, what is the value of x when $y = 1.25$?

(e) If l is inversely proportional to $\sqrt{m}$ and $l = 6$ when $m = 4$. What is the value of m when $l = 4$?

2. Determine from the values of x and y given below whether they vary directly, inversely or in neither of these ways.

(i)

x	12	3	27	6	2
y	9	36	4	18	54

(ii)

x	4	20	30	40	60	80
y	2	8	10	15	20	25

(iii)

x	2	9	7	4	3
y	10	45	35	20	15

3. If x and y vary inversely and $x = 25$, when $y = 3$. Find y, when $x = 15$.

4. If x and y vary inversely and $y = 45$. Find x when constant of variation = 9.

5. Oranges cost ₹ 54 for three dozens in the super market. What is the cost of eight oranges?

6. A car travels 60 km in 1 hr 30 min. How long will it take to cover a distance of 100 km at the same speed?

7. The extension of an elastic spring is found to vary directly with the weight suspended from it. If a weight of 75 kg produces an extension of 1.4 cm, calculate the weight that would produce an extension of 9.8 cm.

8. In 25 days, the earth picks up 6×10^8 pounds of dust from the atmosphere. How much dust will it pick up in 15 days?

HOTS

While driving his car at a speed of 50 km/hr, Ramit covers a distance from home to his office in 1 hour 30 minutes. One day he was 15 minutes late from his home. In order to reach office at time, what should be the speed of the car?

Also, find the total distance covered by Ramit daily.

You Must Know

1. Two quantities x and y vary directly or are said to be in direct variation if x and y increase or decrease together in such a way that $\frac{x}{y}$ remains constant and is positive, i.e. $\frac{x}{y} = k$ (constant) or $x = ky$.

2. Two quantities x and y vary inversely if an increase in x causes corresponding decrease in y (and vice-versa) in such a way that xy remains constant, i.e. $xy = k$ (positive constant) or $x = \frac{k}{y}$.

3. Speed $= \frac{\text{Distance}}{\text{Time}}$.

4. For problems on the motion of trains,

   (i) time taken by a train to pass a stationary object (like pole, signal post or standing man, etc.) is equal to the time taken by the train to cover a distance equal to the length of the train.

   (ii) time taken by a train of length (x) m to pass a stationary object (like tunnel, bridge, platform, etc.) of length (y) m is equal to the time taken by the train to cover a distance of ($x + y$) m.

APPENDIX

Properties of Proportion

1.	If a : b : : c : d, i.e. $\frac{a}{b}=\frac{c}{d}$, then the given proportion can be written as b : a : : d : c, i.e. $\frac{b}{a}=\frac{d}{c}$ by taking the reciprocals of terms on both sides. This relationship is known as **Invertendo**. e.g. If $\frac{2}{5}=\frac{6}{15}$, then by invertendo $\frac{5}{2}=\frac{15}{6}$
2.	If a : b : : c : d, i.e. $\frac{a}{b}=\frac{c}{d}$, then the given proportion can be written as a : c : : b : d, i.e. $\frac{a}{c}=\frac{b}{d}$ This relationship is known as **Alternendo**. e.g. If $\frac{2}{5}=\frac{6}{15}$, then $\frac{2}{6}=\frac{5}{15}$ by alternendo.
3.	If a : b : : c : d, i.e. $\frac{a}{b}=\frac{c}{d}$, then the given proportion can be written as a + b : b : : c + d : d, i.e. $\frac{a+b}{b}=\frac{c+d}{d}$ This relationship is known as **Componendo**. e.g. If $\frac{2}{5}=\frac{6}{15}$, thus by componendo $\frac{2+5}{5}=\frac{6+15}{15}$, i.e. $\frac{7}{5}=\frac{21}{15}$.
4.	If a : b : : c : d, i.e. $\frac{a}{b}=\frac{c}{d}$, then the given proportion can be written as a – b : b : : c – d : d, i.e. $\frac{a-b}{b}=\frac{c-d}{d}$. This relationship is known as **Dividendo**. e.g. $\frac{5}{2}=\frac{15}{6}$ then by dividendo $\frac{5-2}{2}=\frac{15-6}{6}$, i.e. $\frac{3}{2}=\frac{9}{6}$.

5.	If a : b : : c : d, i.e. $\frac{a}{b}=\frac{c}{d}$, then the given proportion can be written as a + b : a – b : : c + d : c – d, i.e. $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ This relationship is known as **Componendo and Dividendo**. e.g. $\frac{5}{2}=\frac{15}{6}$, then by componendo and dividendo $\frac{5+2}{5-2}=\frac{15+6}{15-6}$ i.e. $\frac{7}{3}=\frac{21}{9}$
6.	If a : b : : c : d : : e : f i.e. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then a : b : : c : d : : e : f : : (a + c+ e) : (b + d + f), i.e. $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{a+c+e}{b+d+f}$ This relationship is known as **Addendo**. e.g. If $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$, then $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{1+2+3}{2+4+6}$ i.e. $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{6}{12}$

CHAPTER 5 PROFIT, LOSS AND DISCOUNT

INTRODUCTION

Do you remember some facts about profit and loss?

Let us recall the concepts of cost price, selling price, profit and loss.

The amount paid to purchase an article or the price at which an article is made, is known as its **cost price** and is abbreviated as C.P. The price at which an article is sold, is called its **selling price** and is abbreviated as S.P.

If the selling price is greater than the cost price, the difference between the selling price and cost price is called **profit**,

i.e. $\text{Profit} = \text{S.P.} - \text{C.P.}$

The **profit per cent** is the profit that would be obtained for a cost price of ₹ 100,

i.e. $$\text{Profit }\% = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

If the selling price of an article is less than the cost price, the difference between the cost price and the selling price is called **loss**,

i.e. $\text{Loss} = \text{C.P.} - \text{S.P.}$

The **loss per cent** is the loss that would be made for a cost price of ₹ 100,

i.e. $$\text{Loss }\% = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

Let us recall some more basic relations regarding profit and loss.

(i) $\text{Profit} = \dfrac{\text{Profit }\% \times \text{C.P.}}{100}$

(ii) $\text{Loss} = \dfrac{\text{Loss }\% \times \text{C.P.}}{100}$

(iii) To find S.P. when C.P. and profit % or loss % are given:

(a) $\text{S.P.} = \text{C.P.} \times \left(\dfrac{100 + \text{Profit }\%}{100}\right)$

(b) $\text{S.P.} = \text{C.P.} \times \left(\dfrac{100 - \text{Loss }\%}{100}\right)$

(iv) To find C.P. when S.P. and profit % or loss % are given:

(a) $\text{C.P.} = \dfrac{\text{S.P.} \times 100}{100 + \text{Profit}\,\%}$ (b) $\text{C.P.} = \dfrac{\text{S.P.} \times 100}{100 - \text{Loss}\,\%}$

Now, let us consider a few examples to revive our memory.

Example 1: A mobile phone is sold for ₹ 3,120 at a loss of 4%. What will be the gain or loss per cent, if it is sold for ₹ 3,640?

Solution: Here, S.P. = ₹ 3,120 Loss = 4%

$\therefore$ $\text{C.P.} = \dfrac{100 \times \text{S.P.}}{100 - \text{Loss}\,\%} = ₹\,\dfrac{100 \times 3{,}120}{100 - 4} = ₹\,\dfrac{100 \times 3{,}120}{96} = ₹\ 3{,}250$

Now, New S.P. = ₹ 3,640

$\therefore$ Gain = S.P. – C.P.

= ₹ 3,640 – ₹ 3,250 = ₹ 390

Hence, $\text{Gain}\ \% = \dfrac{\text{Gain}}{\text{C.P.}} \times 100 = \dfrac{390}{3{,}250} \times 100 = 12\%$

Example 2: By selling 42 oranges, a person loses a sum equal to the selling price of eight oranges. Find the loss per cent.

Solution: Let the S.P. of 1 orange = ₹ 1

$\therefore$ S.P. of 42 oranges = ₹ 1 × 42 = ₹ 42

Loss = S.P. of 8 oranges

= ₹ 1 × 8 = ₹ 8

Hence, C.P. of 42 oranges = S.P. + Loss

= ₹ 42 + ₹ 8 = ₹ 50

$\therefore$ $\text{Loss}\ \% = \dfrac{\text{Loss}}{\text{C.P.}} \times 100 = \dfrac{8}{50} \times 100 = 16\%$

Example 3: Naveen bought some cricket balls at ₹ 250 for four balls and sold them at ₹ 340 for five balls. Find his gain or loss per cent.

Solution: C.P. of 4 balls = ₹ 250

$\therefore$ C.P. of 1 ball = ₹ 250 ÷ 4 = ₹ 62.50

S.P. of 5 balls = ₹ 340

$\therefore$ S.P. of 1 ball = ₹ 340 ÷ 5 = ₹ 68

Here, S.P. > C.P.

$$\therefore \quad \text{Profit} = \text{S.P.} - \text{C.P.}$$

$$= ₹\ 68 - ₹\ 62.50 = ₹\ 5.50$$

$$\text{So, Profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{5.50}{62.50} \times 100$$

$$= \frac{44}{5}\% = 8\frac{4}{5}\%$$

Worksheet 1

1. **By selling a bedsheet for ₹ 640, a shopkeeper earns a profit of 28%. How much did it cost the shopkeeper?**

2. **Rajan purchased 250 packets of blades at the rate of ₹ 8 per packet. He sold 70% of the packets at the rate of ₹ 11 per packet and remaining packets at the rate of ₹ 9 per packet. Find his gain per cent.**

3. **Ankit sold two jeans for ₹ 990 each. On one he gains 10% and on the other he lost 10%. Find his gain or loss per cent in the whole transaction.**

4. **Nidhi purchased two *sarees* for ₹ 2,150 each. She sold one *saree* at a loss of 8% and the other at a gain. If she had a gain of ₹ 1,230 on the whole transaction, find the selling price of the second *saree*.**

5. **By selling 35 greeting cards, a shopkeeper loses an amount equal to the selling price of five greeting cards. Find his loss per cent.**

6. **A man bought bananas at the rate of 10 for ₹ 45 and sold at the rate of one dozen bananas for ₹ 51. Find his gain or loss per cent.**

DISCOUNT

In our daily life whenever we go to the market, we see banners and big hoardings indicating 'sale up to 50%' or 'buy one get one free' or 'buy two shirts and get the third shirt for ₹ 1'. These are different methods to attract the customers. In fact, shopkeeper wants to get maximum price for his goods and the customer tries to pay as less as possible.

When the season changes (winter season or summer season), the shopkeeper offers certain schemes and rebates so that more customers are attracted and his stock gets cleared.

During festival season also, the shopkeepers offer different type of rebates in order to increase the sale. These incentives are offered to make customers believe that they get goods at cheaper prices.

In malls and departmental stores, every item is tagged with a card and its price is written on it which is called **marked price (M.P.)** or **list price (L.P.)**. In order to increase the sale or clear the old stock, sometimes the shopkeepers offer a certain percentage of rebate on the marked price. This rebate is called **discount**.

Thus, Selling price = Marked price – Discount

S.P. = M.P. – Discount ... (i)

or Discount = M.P. – S.P.

Hence, $$\boxed{\text{Discount \%} = \frac{\text{Discount}}{\text{M.P.}} \times 100}$$

or $$\text{Discount} = \frac{\text{M.P.} \times \text{Discount \%}}{100} \quad \text{... (ii)}$$

From (i) and (ii),

$$\text{S.P.} = \text{M.P.} - \frac{\text{M.P.} \times \text{Discount \%}}{100}$$

$$= \text{M.P.} \left[1 - \frac{\text{Discount \%}}{100}\right]$$

$$= \frac{\text{M.P.} \times [100 - \text{Discount \%}]}{100}$$

$\therefore$ $$\boxed{\text{M.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Discount \%}}}$$

Let us consider some examples to explain the concepts of discount and marked price.

Example 4: The marked price of a shirt is ₹ 940 and the shopkeeper allows a discount of 15% on it. Find the discount and the selling price of the shirt.

Solution: M.P. of a shirt = ₹ 940 Rate of discount = 15%

$\therefore$ Discount = 15% of ₹ 940 = ₹ $\frac{15}{100} \times 940$ = ₹ 141

Hence, S.P. of the shirt = M.P. – Discount

= ₹ 940 – ₹ 141 = ₹ 799

Example 5: The price of a refrigerator was slashed from ₹ 35,000 to ₹ 29,400 in the winter season. Find the rate of discount.

Solution: M.P. = ₹ 35,000 S.P. = ₹ 29,400

$\therefore$ Discount = M.P. – S.P.

= ₹ 35,000 – ₹ 29,400 = ₹ 5,600

Hence, $$\text{Discount \%} = \frac{\text{Discount}}{\text{M.P.}} \times 100 = \frac{5,600}{35,000} \times 100 = 16\%$$

Hence, the rate of discount is 16%.

Example 6: After allowing a discount of 8% on a book, it is sold for ₹ 828. Find the marked price of the book.

Solution: Here, S.P. = ₹ 828 Discount = 8%

We know that $$\text{M.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Discount \%}}$$

$$= ₹\ \frac{828 \times 100}{100 - 8} = ₹\ \frac{828 \times 100}{92} = ₹\ 900$$

Hence, the marked price of the book is ₹ 900.

Example 7: The marked price of a T.V. is ₹ 32,500. After allowing a 20% Diwali discount to the customer, a shopkeeper still makes a profit of 30%. Find the cost price of the T.V.

Solution: Here, M.P. of the T.V. = ₹ 32,500 Discount = 20%

$\therefore$ $$\text{Discount} = 20\% \text{ of } ₹\ 32,500 = ₹\ \frac{20}{100} \times 32,500 = ₹\ 6,500$$

Hence, S.P. of the T.V. = M.P. – Discount

= ₹ 32,500 – ₹ 6,500 = ₹ 26,000

Profit = 30%

$\therefore$ $$\text{C.P.} = \frac{100 \times \text{S.P.}}{100 + \text{Profit \%}}$$

$$= ₹\ \frac{100 \times 26,000}{100 + 30} = ₹\ \frac{100 \times 26,000}{130} = ₹\ 20,000$$

Hence, the cost price of a T.V. is ₹ 20,000.

Example 8: A dealer allows a discount of 16% to his customers and still gains 5%. Find the marked price of the table which costs him ₹ 1,200.

Solution: Here, C.P. of a table = ₹ 1,200   Profit = 5%

$\therefore$ Profit = 5% of ₹ 1,200 = ₹ $\frac{5}{100} \times 1,200$ = ₹ 60

Hence, S.P. of a table = C.P. + Profit

= ₹ 1,200 + ₹ 60 = ₹ 1,260

Also, Discount = 16%

$$\therefore \quad \text{M.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Discount \%}}$$

$$= ₹\frac{1,260 \times 100}{100 - 16} = ₹\frac{1,260 \times 100}{84} = ₹\ 1,500$$

Hence, the marked price of a table is ₹ 1,500.

Example 9: A shopkeeper allows 25% discount on the marked price of the *sarees* and still makes a profit of 20%. If he gains ₹ 225 over the sale of one *saree*, find the marked price of the *saree*.

Solution: Let the M.P. of one *saree* = ₹ 100   Discount = 25%

$$\therefore \quad \text{Discount} = \frac{\text{Discount \%}}{100} \times \text{M.P.} = ₹\frac{25}{100} \times 100 = ₹\ 25$$

Hence, S.P. of one *saree* = M.P. – Discount = ₹ 100 – ₹ 25 = ₹ 75

Profit = 20%

$$\therefore \quad \text{C.P. of one } saree = \frac{\text{S.P.} \times 100}{100 + \text{Profit \%}} = ₹\frac{75 \times 100}{100 + 20}$$

$$= ₹\frac{75 \times 100}{120} = ₹\ 62.50$$

Thus, Gain = S.P. – C.P. = ₹ 75 – ₹ 62.50 = ₹ 12.50

Now, If the gain is ₹ 12.50, then M.P. = ₹ 100

If the gain is ₹ 1, then M.P. = ₹ $\frac{100}{12.5}$

$\therefore$ If the gain is ₹ 225, then M.P. = ₹ $\frac{100}{12.5} \times 225$

$$= ₹\,\frac{100 \times 225 \times 10}{125} = ₹\ 1{,}800$$

Hence, the marked price of a *saree* is ₹ 1,800.

Alternative Method:

$$\text{Discount} = 25\% \qquad \text{(Given)}$$

$$\text{Profit} = 20\% \qquad \text{(Given)}$$

$$\text{Gain} = ₹\ 225 \qquad \text{(Given)}$$

$$\Rightarrow \quad 20\% \text{ of C.P.} = ₹\ 225$$

$$\Rightarrow \quad \frac{20}{100} \times \text{C.P.} = ₹\ 225$$

$$\Rightarrow \quad \text{C.P.} = ₹\ \frac{22 \times 100}{20} = ₹\ 1{,}125$$

Now,

$$\text{S.P.} = \text{C.P.} + \text{Gain}$$

$$= ₹\ 1{,}125 + ₹\ 225$$

$$= ₹\ 1{,}350$$

$$\text{M.P.} = \frac{\text{S.P.} \times 100}{100 - \text{Discount\%}}$$

$$= ₹\ \frac{1{,}350 \times 100}{100 - 25}$$

$$= ₹\ \frac{1{,}350 \times 100}{75}$$

$$= ₹\ 1{,}800$$

Hence, the marked price of a *saree* is ₹ 1,800.

Worksheet 2

1. **The marked price of a pant is ₹ 1,250 and the shopkeeper allows a discount of 8% on it. Find the discount and the selling price of the pant.**

2. **The marked price of a water cooler is ₹ 5,400. The shopkeeper offers an off season discount of 20% on it. Find its selling price.**

3. **An almirah of marked price ₹ 4,000 is sold for ₹ 3,700 after allowing certain discount. Find the rate of discount.**

4. **Find the rate of discount being given on a ceiling fan whose selling price is ₹ 1,175 after allowing a discount of ₹ 75 on its marked price.**

5. **Find the marked price of a washing machine which is sold at ₹ 8,400 after allowing a discount of 16%.**

6. **A dinner set was bought for ₹ 2,464 after getting a discount of 12% on its marked price. Find the marked price of the dinner set.**

7. **The marked price of a computer is ₹ 22,000. After allowing a 10% discount, a dealer still makes a profit of 20%. Find the cost price of a computer.**

8. **The marked price of a double bed is ₹ 9,575. A shopkeeper allows a discount of 12% and still gains 10%. Find the cost price of the double bed.**

9. **A dealer buys a bicycle for ₹ 1,250 and marks it at 40% above its cost price. If he allows 8% discount, find**
   (i) Selling pice of the bicycle
   (ii) Profit percentage

10. **Priti allows 8% discount on the marked price of the suits and still makes a profit of 15%. If her gain over the sale of a suit is ₹ 156, find the marked price of a suit.**

VALUE ADDED TAX (VAT)

Let us have a look at the bill given below:

Bill No. 085		**MOHIT STORES** Kamla Nagar, Delhi		Date 13-05-16
S.No.	**Item**	**Quantity**	**Rate**	**Amount**
1	00785	1 VAT 5%	₹ 1,000	₹ 1,000 + ₹ 50
Total				₹ 1,050

What do you think does **Value Added Tax (VAT)** mean?

VAT is a tax on estimated market value, added to the value of a product or material at each stage of its manufacture or distribution, which is ultimately passed on to the customer.

For example, when a T.V. set is built by a manufacturer, VAT is charged on all the supplies they purchase for assembling the T.V. Once the T.V. reaches the shelf, the customer who purchases it must pay the VAT that applies him or her.

Let us look at the following examples to understand VAT.

Example 10: Meera bought an A.C. for ₹ 22,000 including a VAT of 10%. Find the price of the A.C. before VAT was added.

Solution: Let the price of an A.C. = ₹ 100

$$\text{VAT on A.C.} = 10\% \text{ of } 100$$

$$= \frac{10}{100} \times 100 = ₹\ 10$$

Total money paid for A.C. = ₹ 100 + ₹ 10 = ₹ 110

If the price paid for an A.C. is ₹ 110, then cost price of A.C. = ₹ 100

If the price paid for an A.C. is ₹ 1, then cost price of A.C. $= ₹\ \frac{100}{110}$

If the price paid for an A.C. is ₹ 22,000, then price of A.C. $= ₹\ \frac{100}{110} \times 22{,}000$

∴ Cost Price of the A.C. = ₹ 20,000

Alternative Method:

Let price of the A.C. before VAT was added = ₹ x

Now, Cost price + VAT = ₹ 22,000

$$x + 10\% \text{ of } x = ₹\ 22{,}000$$

$$\Rightarrow \quad x + \frac{10}{100}x = ₹\ 22{,}000$$

$$\Rightarrow \quad \frac{110}{100}x = ₹\ 22{,}000$$

$$\Rightarrow \qquad x = ₹\ \frac{22,000 \times 100}{110} = ₹\ 20,000$$

$\therefore$ Price of the A.C before VAT was added is ₹ 20,000.

Example 11: Anuj buys a pair of Nike shoes for ₹ 6,000 and VAT charged on it is 7%. Find the amount that Anuj pays for the shoes.

Solution: Cost of a pair of shoes = ₹ 6,000 VAT charged = 7% of 6,000

$$\text{VAT amount} = ₹\ \frac{7}{100} \times 6,000$$

$$= ₹\ 420$$

$\therefore$ Total amount paid by Anuj

$$= ₹\ (6,000 + 420)$$

$$= ₹\ 6,420$$

Worksheet 3

1. **Rehana purchased a dress for ₹ 5,400 including 8% VAT. Find the price of the dress before VAT was added.**

2. **Find the amount to be paid for the following items if 5% VAT is charged on each item.**

   (i) A bottle of hair styling gel at ₹ 185. (ii) A laptop at ₹ 55,000.

   (iii) A mobile at ₹ 36,200.

3. **Raman purchased a music system at ₹ 48,000 including VAT. If the cost pice of the music system was ₹ 40,000, how much VAT (in %) has he paid?**

Value Based Question

Mohit went to a shop and bought a shirt of ₹ 1,650 including 10% VAT. The shopkeeper did not give receipt to him.

(a) Find the amount of VAT not paid by the shopkeeper to the Government.

(b) Is tax evasion correct? Will you tolerate this?

(c) Why it is important to pay tax?

Brain Teasers

1.A. Tick (✓) the correct option.

(a) If the selling price of an article is twice the cost price, the profit per cent is—

(i) 50% (ii) 100% (iii) 150% (iv) 200%

(b) A jeans is marked for ₹ 2,590, but is sold for ₹ 2,331, then discount % is—

(i) 20% (ii) 15% (iii) 10% (iv) 5%

(c) If selling price of five pens is equal to the cost price of four pens, then the gain or loss% is—

(i) 20% gain (ii) 20% loss (iii) 25% loss (iv) 25% gain

(d) Selling price of a *saree* is ₹ 864 including 8% VAT. The original price of the *saree* is—

(i) ₹ 842 (ii) ₹ 800 (iii) ₹ 801.50 (iv) ₹ 820

(e) Discount is always calculated on—

(i) cost price (ii) marked price (iii) selling price (iv) VAT

B. Answer the following questions.

(a) After giving a discount of 8% on the marked price, an article was sold for ₹ 414. Find the marked price of the article.

(b) A fan is sold for ₹ 650. The gain is one-fourth of the cost price of the fan. Find the gain per cent.

(c) A shopkeeper buys pencils at 10 for ₹ 10 and sells them at 8 for ₹ 10. Find the profit per cent.

(d) Find the rate of VAT if an article marked at ₹ 5,000 is sold for ₹ 5,200?

(e) A person pays ₹ 2,800 for a cooler marked at ₹ 3,500. Find the discount per cent offered.

2. Rajan purchased a purse at 25% discount on its marked price but sold it at the marked price. Find the gain per cent of Rajan on this transaction.

3. Jasleen marks her goods at 30% above the cost price and allows a discount of 25% on the marked price. Find her gain or loss per cent.

4. **How much per cent above the cost price should a shopkeeper mark his goods so that after allowing a discount of 20% on the marked price, he gains 12%?**

5. **Rohit marks his goods at 40% above the cost price but allows a discount of 5% for cash payment to his customers. What actual profit does he make, if he receives ₹ 1,064 after allowing the discount?**

6. **Mr Kumar went to shopping with his family to a Mall. Mrs Kumar bought a *saree* for ₹ 12,500, clothes for the kids for ₹ 9,280 and a mobile for Mr Kumar for ₹ 32,638. If the VAT charged on their purchases is 7.5%, then what is the total amount that Mr Kumar has paid?**

HOTS

The marked price of an article is ₹ 3,500 and rate of VAT is 8%. A shopkeeper allows a discount of 20% and still makes a profit of 10%. Find the original cost price of the article and the selling price including VAT.

You Must Know

1. The price marked on an article is called marked price.
2. A certain percentage of rebate on the marked price is called discount.
3. Discount = M.P. – S.P.
4. Discount % = $\dfrac{\text{Discount}}{\text{M.P.}} \times 100$
5. S.P. = $\dfrac{\text{M.P.} \times (100 - \text{Discount \%})}{100}$
6. M.P. = $\dfrac{\text{S.P.} \times 100}{100 - \text{Discount \%}}$
7. VAT is charged on the cost price of an article. It is calculated as

   Amount of VAT $= \dfrac{\text{VAT \%}}{100} \times \text{C.P.}$

   and it is added to the C.P. of an article.

CHAPTER 6

COMPOUND INTEREST

INTRODUCTION

In Class-VII, we have learnt about simple interest. Let us recall the formula for calculating Simple Interest.

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

Here, **P** is the **Principal** (amount), **R** is the **Rate of interest per annum** and **T** is the **Time period in years**.

We know that when money is borrowed on simple interest, the interest is calculated uniformly on the actual principal throughout the loan period. But banks, post offices, insurance corporations and other financial institutions use different method of computing interest. In this method, the borrower and the lender agree to fix up a certain time interval, say, one year or a half-year or one quarter of a year, i.e. three months for the computation of interest and amount. At the end of first time interval, the interest is calculated, and, is added to the original principal. The amount so obtained is taken as the principal for the second time interval. The amount at the end of second time interval is taken as the principal for the third time interval. This process can be repeated for specified periods. After the certain specified period, the difference between the amount at the end of the last period and the original principal is called the **Compound Interest** (abbreviated as **C.I.**). The time period after which interest is calculated and then added to the principal each time to form a new principal is called the **Conversion Period**. As we have discussed earlier, the conversion period may be one year, half-year, three months or one month, and the interest is said to be compounded annually, semi-annually, quarterly or monthly respectively.

COMPUTATION OF COMPOUND INTEREST

There are different methods for calculating the amount and the compound interest. To understand the concept of compound interest, we first take up an example of simple interest.

Let us suppose, we have to calculate interest on a sum of ₹ 10,000 at 6% per annum simple interest for two years.

We know that—

$$\text{Simple Interest} = \frac{P \times R \times T}{100}$$

$$= ₹\ \frac{10{,}000 \times 6 \times 2}{100}$$

$$= ₹\ 1{,}200$$

Now, consider a different situation.

Arjika borrows a sum of ₹ 10,000 from a bank for one year at the rate of 6% per annum. Then at the end of one year, Arjika has to pay back the principal and the interest on ₹ 10,000 for one year at the rate of 6% per annum. Therefore, total amount to be paid by Arjika to the bank shall be—

$$= ₹\ 10{,}000 + \frac{10{,}000 \times 6 \times 1}{100}$$

$$= ₹\ 10{,}000 + 600$$

$$= ₹\ 10{,}600$$

Suppose, Arjika is not in a position to pay this amount to the bank. Then, the bank will charge the interest on ₹ 10,600 thereafter.

At the end of the second year, Arjika has to pay the new principal, i.e. ₹ 10,600 and the interest on ₹ 10,600 for one year at the rate of 6% per annum, i.e. ₹ $\frac{10{,}600 \times 6 \times 1}{100}$ = ₹ 636.

Therefore, the total amount to be paid by Arjika to the bank—

$$= ₹\ 10{,}600 + ₹\ 636$$

$$= ₹\ 11{,}236$$

Hence, the total interest payable to the bank

$$= ₹\ 11{,}236 - ₹\ 10{,}000$$

$$= ₹\ 1{,}236$$

The interest calculated in this manner is called **compound interest**. Therefore, we can say that the compound interest on ₹ 10,000 for two years at the rate of 6% per annum is ₹ 1,236.

The simple interest on ₹ 10,000 for the same time, i.e. two years at the same rate of interest, i.e. 6% per annum shall be ₹ 1,200 while the compound interest is ₹ 1,236 which is ₹ 36 more than the simple interest. The main difference between simple interest and compound interest is that, in case of simple interest, the principal remains the same throughout, whereas in case of compound interest, the principal goes on changing periodically. In this case, the interest is calculated on the original principal and the interest at the end of the first conversion period.

Let us consider a few examples.

Example 1: Find the compound interest on ₹ 15,000 for two years at 8% per annum.

Solution: **For the first year:**

Here, Principal = ₹ 15,000

Time = 1 year

Rate of Interest = 8% p.a.

$\therefore$ Simple Interest $= \dfrac{P \times R \times T}{100}$

$= ₹\ \dfrac{15{,}000 \times 1 \times 8}{100}$

= ₹ 1,200

$\therefore$ Interest for the first year = ₹ 1,200

So, Amount = P + S.I.

= ₹ 15,000 + ₹ 1,200

= ₹ 16,200

For the second year:

In the second year, the amount in the first year becomes the principal here.

So we have, Principal = ₹ 16,200

Time = 1 year

Rate of Interest = 8% p.a.

$\therefore$ Simple Interest $= ₹\ \dfrac{16{,}200 \times 1 \times 8}{100}$

= ₹ 1,296

So, Compound Interest for two years = ₹ 1,200 + ₹ 1,296 = ₹ 2,496

Example 2: Find the amount and compound interest on ₹ 12,000 for three years at 20% per annum.

Solution: **For the first year:**

Here, Principal = ₹ 12,000

Time = 1 year

$$\text{Rate of Interest} = 20\% \text{ p.a.}$$

$$\therefore \quad \text{Simple Interest} = \frac{P \times R \times T}{100}$$

$$= ₹ \frac{12{,}000 \times 1 \times 20}{100}$$

$$= ₹ 2{,}400$$

$$\therefore \text{ Interest for the first year} = ₹ 2{,}400$$

So $$\text{Amount} = P + S.I.$$

$$= ₹ 12{,}000 + ₹ 2{,}400$$

$$= ₹ 14{,}400$$

For the second year:

$$\text{Principal} = ₹ 14{,}400$$

$$\text{Time} = 1 \text{ year}$$

$$\text{Rate of Interest} = 20\% \text{ p.a.}$$

$$\therefore \quad \text{Simple Interest} = ₹ \frac{14{,}400 \times 1 \times 20}{100}$$

$$= ₹ 2{,}880$$

$$\therefore \quad \text{Amount} = ₹ 14{,}400 + ₹ 2{,}880 = ₹ 17{,}280$$

For the third year:

$$\text{Principal} = ₹ 17{,}280$$

$$\text{Time} = 1 \text{ year}$$

$$\text{Rate of Interest} = 20\% \text{ p.a.}$$

$$\therefore \quad \text{Simple Interest} = ₹ \frac{17{,}280 \times 1 \times 20}{100}$$

$$= ₹ 3{,}456$$

$$\therefore \text{ Amount at the end of three years} = P + S.I.$$

$$= ₹ 17{,}280 + ₹ 3{,}456$$

$$= ₹ 20{,}736$$

$$\therefore \text{ Compound Interest for three years} = \text{Amount} - \text{Principal}$$

$$= ₹ 20{,}736 - ₹ 12{,}000 = ₹ 8{,}736$$

Example 3: Rajat bought a washing machine on credit. If the washing machine costs ₹ 9,000 and the shopkeeper charges interest at the rate of 6% per annum, calculate the amount and the compound interest Rajat will have to pay after three years.

Solution: Principal for the first year = ₹ 9,000

Interest for the first year = ₹ $\frac{9{,}000 \times 6 \times 1}{100}$

= ₹ 540

Amount at the end of first year = ₹ 9,000 + ₹ 540

= ₹ 9,540 (principal for second year)

∴ Interest for the second year = ₹ $\frac{9{,}540 \times 6 \times 1}{100}$ = ₹ 572.40

Amount at the end of second year = ₹ 9,540 + ₹ 572.40

= ₹ 10,112.40 (principal for third year)

∴ Interest for the third year = ₹ $\frac{10{,}112.40 \times 6 \times 1}{100}$

= ₹ 606.744

= ₹ 606.74

Amount paid by Rajat at the end of third year

= ₹ 10,112.40 + ₹ 606.74

= ₹ 10,719.14

Hence, Compound Interest paid by Rajat

= Amount – Principal

= ₹ 10,719.14 – ₹ 9,000

= ₹ 1,719.14

Therefore, the Amount and the Compound Interest paid by Rajat after three years are ₹ 10,719.14 and ₹ 1,719.14 respectively.

In the above examples, we use the concept of **unitary method** to calculate the compound interest. We can also calculate Compound Interest and Amount directly by the use of formula as explained in the coming pages.

Worksheet 1

1. **Find the compound interest on ₹ 25,000 at the rate of 12% per annum for three years.**

2. **Find the compound interest on ₹ 6,500 for two years at 9% per annum.**

3. **Find the amount and compound interest on a sum of ₹ 8,000 at 5% per annum for three years compounded annually.**

4. **Harvinder deposited ₹ 40,000 in a post office for a period of three years. The post office credits the interest yearly in his account at 7% per annum compounded annually. Find the balance in his account after three years.**

5. **Monika borrowed ₹ 4,096 from Shalini for three years at $6\frac{1}{4}$% per annum. Find the amount and the compound interest paid by her to Shalini after three years if the interest is compounded annually.**

6. **Ravi purchased a house from DDA on credit. If the cost of the house is 7,50,000 and DDA charges interest at 6% per annum compounded annually, find the interest paid by Ravi if he makes payment to DDA after three years.**

Finding Compound Interest When Interest is Compounded Half-Yearly

In such cases,

- The rate of interest is R% p.a., then it is clearly $\left(\frac{R}{2}\right)$% per half-year.
- Time is also changed into the number of half years.
- Amount after first half-year becomes principal for next half-year and so on.

Thus, when interest is compounded half-yearly, the rate R% p.a. become $\frac{R}{2}$% per half-year and '*n*' years becomes '2*n*' half-years.

It will be more clear from the example given below.

Example 4: Compute the compound interest on ₹ 5,000 for $1\frac{1}{2}$ years at 16% per annum, compounded half-yearly.

Solution: Principal for the first half-year = ₹ 5,000

Rate of Interest = 16% per annum

= 8% per half-yearly

$$\text{Time period} = 1\frac{1}{2} \text{ years}$$

$$= 3 \text{ half-years}$$

$\therefore$ Interest for the first half-year = ₹ $\left[\frac{5,000 \times 1 \times 8}{100}\right]$ = ₹ 400

So, Amount at the end of first half-year = ₹ (5,000 + 400) = ₹ 5,400

So, Principal for the second half-year = ₹ 5,400

$\therefore$ Interest for the second half-year = ₹ $\left[\frac{5,400 \times 1 \times 8}{100}\right]$ = ₹ 432

So, Amount at the end of second half-year = ₹ (5,400 + 432) = ₹ 5,832

$\therefore$ Principal for the third half-year = ₹ 5,832

So, Interest for the third half-year = ₹ $\left[\frac{5,832 \times 1 \times 8}{100}\right]$ = ₹ 466.56

Amount at the end of third half-year = ₹ (5,832 + 466.56) = ₹ 6,298.56

$\therefore$ Compound Interest = ₹ (6,298.56 – 5,000) = ₹ 1,298.56

Finding Compound Interest When Interest is Calculated Quarterly

In such cases, if the rate of interest is R% p.a., then for each quarter it becomes $\frac{R}{4}$% and likewise, the amount after first quarter becomes principal for the second quarter; the amount for the second quarter becomes the principal for the third quarter and so on.

And clearly, the time period is also converted into the number of quarters, i.e. *n* years become 4*n* quarters.

The above concept will be more clear from the following example.

Example 5: Find the compound interest on ₹ 8,000 for six months at 20% per annum compounded quarterly.

Solution: Here,

Rate of Interest = 20% p.a.

= 5% per quarter

Time period = 6 months

= 2 quarters

Principal for the first quarter = ₹ 8,000

$\therefore$ Interest for the first quarter = ₹$\left[\frac{8,000\times1\times5}{100}\right]$ = ₹ 400

Amount at the end of first quarter

= ₹ (8,000 + 400) = ₹ 8,400

So, New principal for the second quarter = ₹ 8,400

Interest for the second quarter = ₹$\left[\frac{8,400\times1\times5}{100}\right]$ = ₹ 420

So, Amount at the end of second quarter = ₹ (8,400 + 420) = ₹ 8,820

Hence, Compound Interest = ₹ (8,820 – 8,000)

= ₹ 820

Note: When conversion period is not mentioned in the given problem, it is taken as one year.

Worksheet 2

1. **Compute the compound interest on ₹ 5,000 for $1\frac{1}{2}$ years at 16% per annum compounded half-yearly.**

2. **Find the compound interest on ₹ 15,625 at 16% per annum for nine months when compounded quarterly.**

3. **Rohit deposited ₹ 10,000 in a bank for six months. If the bank pays compound interest at 12% per annum reckoned quarterly, find the amount to be received by him on maturity.**

4. **Find the difference between the compound interest on ₹ 25,000 at 16% per annum for six months compounded half-yearly and quarterly respectively. Which option is better?**

5. **Bela borrowed ₹ 25,000 from a finance company to start her boutique at 20% per annum compounded half-yearly. What amount of money will clear her debt after $1\frac{1}{2}$ years?**

FORMULA FOR FINDING THE COMPOUND INTEREST

The method adopted for calculating compound interest in the previous section is very lengthy and time consuming. Therefore, we should try to find a formula for calculating the compound interest. Suppose, we have to find the compound interest on ₹ P for *n* years at R% per annum compounded annually.

Principal for the first year = ₹ P

$\therefore$ Interest for the first year = ₹ $\frac{P \times R \times 1}{100}$

$= ₹ \frac{PR}{100}$

Hence, Amount at the end of first year

$= ₹ P + ₹ \frac{PR}{100}$

$= ₹ P\left(1+\frac{R}{100}\right)$ (principal for the second year)

$\therefore$ Interest for the second year

$= ₹ \frac{P\left(1+\frac{R}{100}\right) \times R \times 1}{100}$

$= ₹ \frac{PR}{100}\left(1+\frac{R}{100}\right)$

Hence, Amount at the end of second year

$= ₹ P\left(1+\frac{R}{100}\right) + ₹ \frac{PR}{100}\left(1+\frac{R}{100}\right)$

$= ₹ P\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$

$= ₹ P\left(1+\frac{R}{100}\right)^2$ (principal for the third year)

$\therefore$ Interest for the third year = ₹ $\frac{P\left(1+\frac{R}{100}\right)^2 \times R \times 1}{100}$

$$= ₹\frac{PR}{100}\left(1+\frac{R}{100}\right)^2$$

Hence, Amount at the end of third year $= ₹P\left(1+\frac{R}{100}\right)^2 + ₹\frac{PR}{100}\left(1+\frac{R}{100}\right)^2$

$$= ₹P\left(1+\frac{R}{100}\right)^2\left[1+\frac{R}{100}\right]$$

$$= ₹P\left(1+\frac{R}{100}\right)^3$$

If we proceed in the same manner, we see that–

Amount at the end of n years $= ₹P\left(1+\frac{R}{100}\right)^n$

Hence, Compound Interest for n years

$$= \text{C.I.} = \text{Amount} - \text{Principal}$$

$$= P\left(1+\frac{R}{100}\right)^n - P$$

$$= P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

Thus, $\boxed{A = P\left(1+\frac{R}{100}\right)^n}$ and $\boxed{\text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]}$

where, P is the principal

R is the rate of interest per annum

n is the number of conversion period (years in the present case)

Let us consider a few examples to illustrate the use of the above formulae.

Example 6: Find the amount for ₹ 62,500 lent at 4% per annum for two years, compounded annually.

Solution: We know that—

$$A = P\left(1+\frac{R}{100}\right)^n$$

Here, P = ₹ 62,500, R = 4, $n = 2$, A = ?

$$\therefore \qquad A = ₹\ 62{,}500\left(1+\frac{4}{100}\right)^2$$

$$= ₹\ 62{,}500\left(1+\frac{1}{25}\right)^2 = ₹\ 62{,}500\left(\frac{26}{25}\right)^2$$

$$= ₹\ 62{,}500 \times \frac{26}{25}\times\frac{26}{25} = ₹\ 67{,}600$$

Therefore, the required amount is ₹ 67,600.

Example 7: Find the amount and compound interest on ₹ 33,280 for three years if the rate of interest is $12\frac{1}{2}\%$ p.a. compounded annually.

Solution: Here, $P = 33{,}280,\ R = 12\frac{1}{2} = \frac{25}{2},\ n = 3,\ A = ?$

We know that—

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= ₹\ 33{,}280\left(1+\frac{25}{200}\right)^3 = ₹\ 33{,}280\left(1+\frac{1}{8}\right)^3$$

$$= ₹\ 33{,}280\left(\frac{9}{8}\right)^3$$

$$A = ₹\ 33{,}280 \times \frac{9}{8}\times\frac{9}{8}\times\frac{9}{8}$$

$$= ₹\ 47{,}385$$

$$\therefore \qquad \text{Compound Interest} = \text{Amount} - \text{Principal}$$

$$= ₹\ 47{,}385 - 33{,}280$$

$$= ₹\ 14{,}105$$

Hence, the required amount is ₹ 47,385 and compound interest is ₹ 14,105.

When Interest is Compounded Half-Yearly

Let Principal = P

Rate of Interest = $\frac{R}{2}\%$ per half-year

Time (n) = $2n$ half-years.

So, $$\text{Amount} = P\left[1+\frac{R/2}{100}\right]^{2n}$$

or $$A = P\left[1+\frac{R}{200}\right]^{2n}$$

Let us look at the following examples.

Example 8: Compute the amount of ₹ 65,536 for $1\frac{1}{2}$ years at $12\frac{1}{2}$% per annum, the interest being compounded semi-annually.

Solution: Here,

Principal = ₹ 65,536

Time = $1\frac{1}{2}$ years = 3 half-years

Rate of Interest = $12\frac{1}{2}$% p.a. = $\left[\frac{25}{4}\right]$% per half-year

$$\therefore \quad \text{Amount} = ₹\ 65{,}536\times\left\{1+\frac{25}{400}\right\}^3$$

$$= ₹\ 65{,}536\times\left\{\frac{17}{16}\right\}^3 = ₹\,65{,}536\times\frac{17}{16}\times\frac{17}{16}\times\frac{17}{16}$$

$$= ₹\ 78{,}608$$

Example 9: Find the compound interest on ₹ 1,000 for 18 months at 10% per annum compounded half-yearly.

Solution: Here,

Principal = ₹ 1000

Rate of Interest = 10% p.a. = 5% half-year

Time = 18 months = 3 half-years

$$\therefore \quad \text{Amount} = ₹\ 1{,}000\left\{1+\frac{5}{100}\right\}^3 = ₹\,1{,}000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}$$

$$= ₹\ \frac{9{,}261}{8} = ₹\ 1{,}157.625 = ₹\ 1{,}157.63$$

$$\therefore \quad \text{C.I.} = A - P$$

$$= ₹\ (1{,}157.63 - 1{,}000)$$

$$= ₹\ 157.63$$

When Interest is Compounded Quarterly

Let Principal = P

Rate of Interest = R% p.a.

$= \frac{R}{4}\%$ per quarter

Time (n) = $4n$ quarter years

Then, $A = P \times \left[1 + \frac{R/4}{100}\right]^{4n}$

or $= P \times \left[1 + \frac{R}{400}\right]^{4n}$

and C.I. = A – P

Example 10: Calculate the compound interest on ₹ 24,000 for six months if the interest is payable quarterly at the rate of 8% per annum.

Solution: Here, Principal = ₹ 24,000

Rate of Interest = 8% p.a = 2% per quarter

Time = 6 months = 2 quarters

$\therefore$ $A = P\left[1 + \frac{R}{100}\right]^{n}$

$= ₹\ 24{,}000\left[1 + \frac{2}{100}\right]^{2}$

$= ₹\ 24{,}000 \times \frac{51}{50} \times \frac{51}{50}$

= ₹ 24,969.60

$\therefore$ C.I. = A – P

= ₹ (24,969.60 – 24,000) = ₹ 969.60

Example 11: A certain sum amounts to ₹ 12,167 in three years at 15% per annum compounded annually. Find the sum.

Solution: Here, $A = ₹\ 12{,}167,\ \ R = 15\%$ p.a., $\ n = 3,\ \ P = ?$

We know that $$A = P\left(1+\frac{R}{100}\right)^n$$

$$₹\ 12{,}167 = P\left(1+\frac{15}{100}\right)^3 = P\left(1+\frac{3}{20}\right)^3$$

$$₹\ 12{,}167 = P\left(\frac{23}{20}\right)^3 = P\times\frac{23}{20}\times\frac{23}{20}\times\frac{23}{20}$$

$$\therefore \quad P = ₹\,\frac{12{,}167\times 20\times 20\times 20}{23\times 23\times 23} = ₹\ 8{,}000$$

Hence, the required sum is ₹ 8,000.

Example 12: At what rate per cent will a sum of ₹ 3,125 amount to ₹ 3,645 in two years?

Solution: Here, $A = ₹\ 3{,}645,\ \ P = ₹\ 3{,}125,\ \ n = 2,\ \ R = ?$

We know that $$A = P\left(1+\frac{R}{100}\right)^n$$

$$₹\ 3{,}645 = ₹\ 3{,}125\left(1+\frac{R}{100}\right)^2$$

$$\frac{3{,}645}{3{,}125} = \left(1+\frac{R}{100}\right)^2$$

$$\frac{729}{625} = \left(1+\frac{R}{100}\right)^2$$

$$\left(\frac{27}{25}\right)^2 = \left(1+\frac{R}{100}\right)^2$$

$$\therefore \quad 1+\frac{R}{100} = \frac{27}{25}$$

$$\frac{R}{100} = \frac{27}{25} - 1$$

$$\frac{R}{100} = \frac{2}{25}$$

$$R = \frac{2}{25} \times 100$$

$$= 8\%$$

Hence, the rate of interest is 8% per annum.

Example 13: In what time will ₹ 5,400 amount to ₹ 6,773.76 at 12% per annum compound interest?

Solution: Here, P = ₹ 5,400, A = 6,773.76, R = 12% p.a., n = ?

We know that $$A = P\left(1+\frac{R}{100}\right)^n$$

$$₹\ 6,773.76 = ₹\ 5,400\left(1+\frac{12}{100}\right)^n$$

$$\frac{6,773.76}{5,400} = \left(1+\frac{3}{25}\right)^n$$

$$\frac{6,77,376}{5,40,000} = \left(\frac{28}{25}\right)^n$$

$$\therefore \quad \left(\frac{28}{25}\right)^n = \frac{784}{625}$$

$$\left(\frac{28}{25}\right)^n = \left(\frac{28}{25}\right)^2$$

$$\therefore \quad n = 2$$

Hence, the given sum of ₹ 5,400 will amount to ₹ 6,773.76 at 12% in two years.

Example 14: The simple interest on a certain sum of money for three years at 5% per annum is ₹ 540. What will be the compound interest on that sum at the same rate for the same period?

Solution: Here, S.I. = ₹ 540, R = 5, T = 3, P = ?

We know that

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

$$₹\ 540 = \frac{P \times 5 \times 3}{100}$$

$$\therefore \quad P = ₹\ \frac{540 \times 100}{5 \times 3}$$

$$= ₹\ 3{,}600$$

Now,

$$\text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

$$= ₹\ 3{,}600\left[\left(1+\frac{5}{100}\right)^3 - 1\right] = ₹\ 3{,}600\left[\left(1+\frac{1}{20}\right)^3 - 1\right]$$

$$= ₹\ 3{,}600\left[\left(\frac{21}{20}\right)^3 - 1\right] = ₹\ 3{,}600\left[\frac{9{,}261}{8{,}000} - 1\right]$$

$$= ₹\ 3{,}600\left[\frac{9{,}261 - 8{,}000}{8{,}000}\right] = ₹\ 3{,}600 \times \frac{1{,}261}{8{,}000}$$

$$= ₹\ 567.45$$

Therefore, the compound interest will be ₹ 567.45.

Example 15: The difference between the compound interest and the simple interest on a certain sum at 10% per annum for three years is ₹ 93. Find the sum.

Solution: Let the sum be ₹ 100.

Here, R = 10% p.a., $n = 3$

$$\therefore \quad \text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

$$= ₹\ 100\left[\left(1+\frac{10}{100}\right)^3-1\right] = ₹\,100\left[\left(1+\frac{1}{10}\right)^3-1\right]$$

$$= ₹\ 100\left[\left(\frac{11}{10}\right)^3-1\right] = ₹\,100\left[\frac{1,331}{1,000}-1\right]$$

$$= ₹\ 100\left[\frac{1,331-1,000}{1,000}\right] = ₹\,100\times\frac{331}{1,000}$$

$$= ₹\ 33.10$$

Also, $$\text{S.I.} = \frac{\text{P}\times\text{R}\times n}{100}$$

$$= ₹\ \frac{100\times10\times3}{100} = ₹\ 30$$

$\therefore$ The difference between C.I. and S.I.

$$= ₹\ 33.10 - ₹\ 30$$

$$= ₹\ 3.10$$

Now, if the difference between C.I. and S.I. is ₹ 3.10, then the sum = ₹ 100

If the difference between C.I. and S.I. is ₹ 1, then the sum = $₹\,\frac{100}{3.10}$

If the difference between C.I. and S.I. is ₹ 93, then the sum = $₹\,\frac{100}{3.10}\times93 = ₹\,3,000$

Hence, the required sum is ₹ 3,000.

Alternative Method:

Let the sum be ₹ P

$$\text{R} = 10\%\text{ p.a.},\ \ n = 3$$

$$\text{S.I.} = \frac{\text{P}\times\text{R}\times n}{100}$$

$$= ₹\ \frac{\text{P}\times10\times3}{100}$$

$$= ₹\,\frac{3\cancel{0}}{10\cancel{0}}\text{P} = ₹\,\frac{3}{10}\text{P}$$

$$\text{C.I.} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$$

Also,

$$= P\left[\left(\frac{11}{10}\right)^3 - 1\right]$$

$$= P\left(\frac{11^3 - 10^3}{10^3}\right)$$

$$= P\left(\frac{1,331 - 1,000}{1,000}\right)$$

$$= ₹\,\frac{331}{1,000}P$$

As given, $\text{C.I.} - \text{S.I.} = ₹\ 93$

i.e. $₹\,\frac{331}{1,000}P - ₹\,\frac{3}{10}P = ₹\ 93$

i.e. $₹\left(\frac{331-300}{1,000}\right)P = ₹\ 93$

i.e. $₹\,\frac{31}{1,000}P = ₹\ 93$

$\therefore$ $P = ₹\,\frac{93 \times 1,000}{31}$

$= ₹\ 3,000$

$\therefore$ The required sum is ₹ 3,000

Worksheet 3

1. Find the amount for ₹ 15,000 at 8% per annum compounded annually for two years.

2. Find the compound interest on ₹ 11,200 at $17\frac{1}{2}$% per annum for two years.

3. Ram borrowed a sum of ₹ 30,000 from Shyam for three years. If the rate of interest is 6% per annum compounded annually, find the interest paid by Ram to Shyam after three years.

4. Nidhi deposited ₹ 7,500 in a bank which pays her 4% interest per annum compounded annually. Find the amount and the interest received by her after three years.

5. Find the difference between the compound interest and the simple interest on ₹ 30,000 at 7% per annum for three years.

6. Aman borrows ₹ 14,500 at 11% per annum for three years at simple interest and Tarun borrows the same amount at 10% per annum for the same time compounded annually. Who pays more interest and by how much?

7. The simple interest on a certain sum of money for two years at $5\frac{1}{2}$% is ₹ 6,600. What will be the compound interest on that sum at the same rate for the same time period?

8. A certain sum amounts to ₹ 2,970.25 in two years at 9% per annum compounded annually. Find the sum.

9. On what sum will the compound interest at $7\frac{1}{2}$% per annum for three years compounded annually be ₹ 3,101.40?

10. At what rate per cent will a sum of ₹ 640 be compounded to ₹ 774.40 in two years?

11. At what rate per cent will a sum of ₹ 64,000 be compounded to ₹ 68,921 in three years?

12. In how many years will ₹ 8,000 amount to ₹ 9,261 at 5% per annum compounded annually?

13. In what time will a sum of ₹ 3,750 at 20% per annum compounded annually amount to ₹ 6,480?

14. The difference between the compound interest and simple interest on a certain sum of money at $6\frac{2}{3}$% per annum for three years is ₹ 46. Find the sum.

15. The difference between the compound interest and the simple interest on a certain sum of money at 15% per annum for three years is ₹ 283.50. Find the sum.

16. Find the amount and the compound interest on ₹ 12,800 for one year at $7\frac{1}{2}$ % per annum compounded semi-annually.

17. Mr Arora borowed ₹ 40,960 from a bank to start a play school. If the bank charges $12\frac{1}{2}$ % per annum compounded half-yearly, what amount will he have to pay after $1\frac{1}{2}$ years?

18. Meera lent out ₹ 20,000 for nine months at 20% per annum compounded quarterly to Mrs Sharma. What amount will she get after the expiry of the period?

19. Find the amount and the compound interest on ₹ 24,000 for six months if the interest is payable quarterly at the rate of 20 paise a rupee per annum.

GROWTH AND DEPRECIATION

Some quantities, such as population, weight, height of a human being increase over a period of time under normal conditions. The relative increase is called **growth**.

Growth per unit of time is called the **rate of growth**.

Let us find the formula for Growth of Population.

Let P be the population at the begining of a year.

Case-I If the population growth is constant for all the given number of years, then

population after n years $= P\left(1+\frac{R}{100}\right)^n$

Case-II If the population growth variates for the given number of years.

Let it be, say, a % for the first year; b % for the second year, then population after two years $= P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)$

Case-III If the population decreases constantly for the given number of (n) years by R%, then population after n years $= P\left[1-\frac{R}{100}\right]^n$

Let us understand this with the help of some examples.

Example 16: The present population of a town is 28,000. If the population increases at the rate of 5% per annum in the first year and 7% per annum in the second year, find the population after two years.

Solution: Present population = 28,000

Rate of growth in first year = 5% p.a.

Rate of growth in second year = 7% p.a.

$$\text{So Population after two years} = 28,000\left(1+\frac{5}{100}\right)\left(1+\frac{7}{100}\right)$$

$$= 28,000 \times \frac{21}{20} \times \frac{107}{100}$$

$$= 31,458$$

Example 17: The population of a town was 62,500 two years ago. It increased every year at the rate of 4% per annum. Find its present population.

Solution: $$\text{Present population} = 62,500\left(1+\frac{4}{100}\right)^2$$

$$= 62,500 \times \frac{52}{50} \times \frac{52}{50} = 67,600$$

$$\therefore \quad \text{Present population} = 67,600$$

Now, let us see what depreciation is. The value of a machine or a building or any other such article, subject to wear and tear, decreases with time.

Relative decrease in the value of a machine is called its **depreciation**.

Depreciation per unit time is called the **rate of depreciation**.

So if P is the value of machine at a certain time and R % per annum is the constant rate of depreciation for n number of years, then the value of the machine after n years

$$= P\left(1 - \frac{R}{100}\right)^n.$$

Let us look at the following few examples.

Example 18: Rajeev bought a car at ₹ 1,75,000. If its value depreciates at the rate of 20% per annum, what will be its value after three years? Also find the total depreciation.

Solution: Initial value of the car = ₹ 1,75,000

Rate of depreciation = 20% p.a.

Value of the car after three years $= ₹\ 1,75,000\left(1-\frac{20}{100}\right)^3$

$= ₹\ 1,75,000\times\frac{4}{5}\times\frac{4}{5}\times\frac{4}{5}$

= ₹ 89,600

∴ Total depreciation = ₹ (1,75,000 – 89,600)

= ₹ 85,400

Example 19: The value of a refrigerator which was purchased two years ago, depreciates at 12% per annum. If its present value is ₹ 9,680, for how much was it purchased?

Solution: Let C.P. of refrigerator = ₹ 100

Depreciation = 12% p.a.

Then the value of refrigerator after depreciation $= ₹\ 100\left(1-\frac{12}{100}\right)^2$

$= ₹\ 100\left(\frac{22}{25}\right)^2$

$= ₹\ \frac{1,936}{25}$

If the depreciated value of refrigerator is $₹\frac{1,936}{25}$, then the original value

= ₹ 100

If the depreciated value of refrigerator is ₹ 1, then the original value

$= ₹ 100\times\frac{25}{1,936}$

If the depreciated value of refrigerator is ₹ 9,680, then the original value

$= ₹ 100\times\frac{25}{1,936}\times 9,680 = ₹\ 12,500$

Worksheet 4

1. **The population of a town is increasing at the rate of 8% per annum. What will be the population of the town after two years if the present population is 12,500?**

2. **Three years ago, the population of a town was 50,000. If the annual increase during three successive years was at the rate 4%, 5% and 4% per annum respectively, find the present population.**

3. **Madhu bought a house for ₹ 1,31,25,000. If its value depreciates at the rate of 10% per annum, what will be its sale price after three years?**

4. **The profits of a firm were ₹ 72,000 in the year 2014. During the next year, it increased by 7% and it decreased by 5% in the following year. What are the profits of the firm after two years?**

5. **The population of a town is 64,000. If the annual birth rate is 10.7% and the annual death rate is 3.2%, calculate the population after three years.**

   [**Hint**: Net growth rate = (Birth rate – Death rate)%]

6. **In a factory, the production of motor bikes was 40,000 in a particular year, which rose to 48,400 in two years. Find the rate of growth per annum, if it was uniform during two years.**

Value Based Questions

1. **Donating blood is a humanitarian act. As per a survey, 16,000 blood donors are registered with 'Red Cross' in Delhi and this number of donors increases at the rate of 5% every six months.**

   (a) How many donors would be there at the end of $1\frac{1}{2}$ years?

   (b) Why is it important to donate blood?

   (c) What steps would you take to encourage people to donate blood?

2. **A town has a population of 2,50,000. The growth rate of the population of the town is 4% per annum.**

   (a) What would be the population after three years?

   (b) What are the harmful effects of such a rapid increase in the population?

Brain Teasers

1.A. Tick (✓) the correct option.

(a) For a given rate of interest and time, what is more profitable to the depositor?

(i) compound interest (ii) simple interest

(iii) both are equally profitable (iv) cannot be determined

(b) A sum of ₹ 10,000 at 8% per annum for six months compounded quarterly amounts to—

(i) ₹ 400 (ii) ₹ 404 (iii) ₹ 408 (iv) ₹ 10,404

(c) In $A = P\left(1+\frac{R}{100}\right)^n$, A stands for—

(i) time period (ii) principal

(iii) principal + interest (iv) interest

(d) If the number of conversion periods is greater than or equal to 2, then compound interest is—

(i) less than simple interest (ii) greater than simple interest

(iii) less than or equal to simple interest (iv) greater than or equal to simple interest

(e) Reema wants to do a one-year deposit. She should opt for—

(i) a simple interest of 10%

(ii) a compound interest of 10% compounded quarterly

(iii) a compound interest of 10% compounded annually

(iv) a compound interest of 10% compounded half-yearly

B. Answer the following questions.

(a) Find the compound interest on ₹ 1,000 at 10% per annum for two years.

(b) If ₹ 6,000 is deposited for two years at 4% per annum compounded quarterly, then find the time period and rate to compute compound interest.

(c) The value of a machine worth ₹ 5,00,000 depreciates at the rate of 10% every year. In how many years will its value be ₹ 3,64,500?

(d) Find the difference between the compound interest and simple interest on ₹ 5,000 for two years at 5% per annum.

(e) If ₹ 20,000 is deposited for three years at 5% compounded annually, then what will be the principal for the second year?

2. Preeti invested ₹ 50,000 at 8% per annum for three years and the interest is compounded annually. Calculate:

(a) the amount standing to her credit at the end of the second year.

(b) the interest for the third year.

3. A man had ₹ 75,000. He invested ₹ 35,000 in a company which pays him 9% interest per annum and he invested rest of the money in another company which pays him 9.5% interest per annum. Find the total compound interest received by him after two years.

4. A certain sum of money at compound interest becomes ₹ 7,396 in two years and ₹ 7,950.70 in three years. Find the rate of interest.

5. Pooja started a business by investing ₹ 2,00,000. During the first three successive years, she earned a profit of 5%, 8% and 12% per annum respectively. If in each year, the profit was added on the capital at the end of the previous year, calculate her total profit after three years.

6. Mahesh borrowed a certain sum for two years at simple interest from Bhim. Mahesh lent this sum to Vishnu at the same rate for two years compound interest. At the end of two years, Mahesh received ₹ 410 as compound interest but paid ₹ 400 as simple interest. Find the sum and rate of interest.

7. A man invested ₹ 1,000 for three years at 11% simple interest per annum and ₹ 1,000 at 10% compound interest per annum compounded annually for three years. Find which investment is better.

8. M/s Heera Associates let out ₹ 4,00,000 for one year at 16% per annum compounded annually. How much they could earn if the interest is compounded half–yearly?

9. Sirish borrowed a sum of ₹ 1,63,840 at 12.5% per annum compounded annually. On the same day, he lent out the same amount to Sahej at the same rate of interest but compounded half-yearly. Find his gain after two years.

10. **The annual rate of growth in population of a certain city is 8%. If its present population is 1,96,830, what was the population three years ago?**

HOTS

A certain sum of money is invested at the rate of 10% per annum compound interest, the interest compounded annually. If the difference between the interests of third year and first year is ₹ 1,105, find the sum invested.

Enrichment Question

A builder employed 4,000 workers to work on a residential project. At the end of the first year, 10% workers were removed, at the end of the second year, 5% of those working at that time were retrenched. However, to complete the project in time, the number of workers was increased by 15% at the end of the third year. How many workers were working during the fourth year?

You Must Know

1. In case of simple interest, the interest is calculated on the initial principal whereas compound interest is calculated on the principal and the interest on it after every conversion period.
2. The period after which interest is added to the principal is called conversion period and the interest so obtained after a number of conversion periods is called compound interest.
3. If the conversion period is one year, the interest is said to be compounded annually.
4. Amount $A = P\left(1+\frac{R}{100}\right)^n$ and Compound Interest, C.I. $= A - P = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$, where P is the Principal, R is the Rate of interest per annum and n is the Number of Conversion periods (years).
5. Increase in certain quantities over a period of time is called growth.
6. Decrease in the value of some asset over a period of time is depreciation.